Cooperating Through Leaders*

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Abstract

We study games of conflict (among groups or countries) where players can choose to fight or cooperate. We consider games where conflict is detrimental, that is where the average welfare from outcomes in which conflict occurs is smaller than the cooperation outcome. The novelty of our approach is that group choices are made under guidance of leaders who offer proposals to passive followers on the best course of action. *Accountability* of leaders is possible because of ex-post punishment which can be imposed by the groups, when the realized utility is smaller than that implicitly promised. *Competition* among leaders is possible if groups are willing to listen to more than one leader. We prove that in all games the limit outcome is efficient under competition if accountability is sufficiently large.

Keywords: Plural Societies, Polarization, Social Conflict, Accountability, Political Equilibria.

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1. Introduction

We study a symmetric two-by-two game between two groups in which a cooperative action taken by both groups gives a higher utility than when either player takes the alternative action of fighting. The novelty of our approach is that there are leaders who offer proposals on the best course of actions to the groups and promise particular outcomes, and the groups act on the basis of these proposals. Leaders act to influence the outcome of the game because their own utility depends on that outcome. Group members are assumed to be unwilling to invest in acquiring information about the best course of action, hence act as followers: that is, they choose the most promising proposal and punish leaders who fail to deliver on their promises.

The presence of leaders induces a new game between the leaders, in which their utility depends upon the outcome and the punishments issued by the followers. The possibility of groups punishing leaders gives the latter an incentive to make credible promises. Such punishment represents a form of accountability that binds leaders or politicians (see Ferejohn (1986), Maskin and Tirole (2004), Besley and Case (1995), and Besley (2006)).

We consider two types of leaders: *group* leaders, whose utility from the outcome is identical to that of their group; and a *common* leader, whose utility is the average of the utilities of the two groups. Our main finding is that the presence of the common leader transforms the nature of the game. If there are only group leaders the game remains essentially the same as that with no leaders. With a common leader cooperation may obtain also in games like the prisoners dilemma. For cooperation to occur two conditions must be met. First, there must be *competition* among leaders, that is, followers must be willing to listen to many sides, and not just do what their group leaders recommend. Second, there must be *accountability*: bad proposals of leaders must be punished by followers when the realized outcomes are worse than the promised ones. The insight offered by this paper is that competing, accountable leaders enable groups to achieve cooperation with surprisingly high probability. Indeed we show that in the families of games we study (which include the prisoner's dilemma and chicken), in the presence of competition among leaders the equilibrium probability of cooperation converges to one as the size of punishment grows.

We also find that punishments of group and common leaders play different roles. What raises the probability of cooperation is the level of punishment of group leaders, because stronger punishments discourage them from promising ephemeral victories. Higher punishments of the common leader may have a negative impact on the probability which with he proposes cooperation, but the equilibrium, the probability of cooperation is largely independent of the common leader punishment level. Model and main results are presented in Sections 2 to 6.

In using the model to understand conflict we then take the view that conflict is more likely to prevail when the group leaders do not fear high enough punishments and there are no accountable common leaders whom the groups judge capable of making viable cooperation proposals. By contrast when group leaders are held back by large potential punishments and there is an accountable third party that can appeal to both sides we should see mediation and mutually beneficial cooperation. In Section 7 we report on some evidence concerning the frequency with which common leaders have emerged and the extent of their success in bringing cooperation, in the contexts of civil wars and internal politics of specific countries. In the former case we look at interventions by the UN but also other actors. As cases of fruitful internal mediation we take Nelson Mandela, who brought about an end to the conflict between the races in South Africa; in the US Eisenhower, who led a brief era of relatively low polarization before the opposite leaders Nixon and Kennedy again fractured the political system; and in Italy the "technical" governments of of Ciampi, Dini and Monti, whose governments did not include politicians and managed to pass much needed structural reforms.

Our concdpt has similarities with that of correlated equilibria, where better outcomes than in Nash equilibria may be reached through the intervention of an external mediator. The differences between games among leaders and correlated equilibria are actually deeper than the similarities: we develope this comparison in Section 8. Section 9 concludes.

1.1. Literature Review

There are other studies where delegation and/or leadership has a role. In Eliaz and Spiegler (2020), as here, a representative agent chooses among policy proposals and then selects and implements the one with the highest expected payoff. The difference is that we explicitly model the proposers and their incentives and allow each of them to address several representative followers. In the tradition of Barro (1973) (and see Miquel (2007) for an application to divided societies), Baliga et al. (2011) develop a model of conflict between countries related to our conflict game, (see also Baliga and Sjöström (2004) and Baliga and Sjöström (2020)). Individuals in the countries (groups) have different payoffs, and may be hawkish (the aggressive action is dominant) or dovish (the accommodating strategy is dominant). There are leaders who choose strategies, and citizens retrospectively support or not the leader, depending on whether the action of the leader was a best response to that of the opponent from their point of view. The main difference with our approach is that the choice in our model is made by the citizens, not by the leader; the latter can only influence the choice of the citizens with their proposals.

Like us, Dutta et al. (2018) consider punishment of leaders, but their punishment is based *ex ante* considerations and there are no common leaders. In the tradition of games with common agency (Dixit et al. (1997)), Prat and Rustichini (2003) explore the idea that games among principals can be played through the mediation of agents who receive transfers conditional on the action chosen, to induce them to play one action rather than another. The setup is different from the one used here, where the direct utility of leaders and followers may be the same, and defined on outcomes, with no transfers; and leaders can be punished so their overall payoff may differ from that of the followers.

The issue of polarization and potential conflict among groups has acquired particular relevance in the period following the second world conflict, as the new post-colonial order emerged. This development was anticipated in the farsighted book by Furnivall (2014) on the development of Burmese society after independence, and the conflicts potentially arising in a multi-ethnic society. Furnivall introduced the key concept of *plural society,* defined as "comprising two or more elements or social orders which live side by side, yet without mingling in one political unit." The concept was further elaborated by Rabushka and Shepsle (1971): "in the plural society - but not in the pluralistic society - the overwhelming preponderance of political conflicts is perceived in ethnic terms." The authors note that this definition "does not explain why some culturally diverse societies are plural and others are not. Typically, however, definitions are not called upon to perform such tasks. What is needed is a theory - a theory, we argue, of political entrepreneurship." Building such a theory is the main purpose of this paper.

Related ideas on polarized society were discussed in Lijphart (1977) and Fearon and Laitin (1996). Papers providing analytical foundations to this idea include Esteban and Ray (1994), Esteban et al. (2012) and Duclos et al. (2004), who construct a general, well founded measure of polarization. The *salience* of ethnic conflict, which was the main parameter marking the transformation from pluralistic to plural society in Rabushka and Shepsle (1971), is analyzed in Esteban and Ray (2008). These models are tested against data in several follow up studies (for example in Esteban et al. (2012), which provides support for the theory). In the context of provision of public goods, a related issue is explored in Alesina et al. (1999); here individuals live in the same city but have different ethnicity and thus heterogeneous preferences on public good; this fragmentation induces inefficiently low provision of public goods.

2. The Game Between Leaders

Interpreting players as large homogeneous groups, we focus on the role of leaders in the collective decision making process. We take the view that, because groups are large, individual members have little incentive to acquire information about the consequences of collective actions, and with limited knowledge of causality they instead listen to leaders and follow the leaders who make them the best offer. Specifically, leaders propose a course of action, implicitly promising a particular outcome if that action is followed. For example, their actions may say "Follow me and fight, the enemy will surrender," or "Follow me and cooperate, the enemy will also cooperate." Followers know their own utility, so they understand that the enemy surrendering is for them a better outcome than war. However, being unwilling to acquire information, they do not speculate about the situation, but simply choose the best offer. After the fact they observe their realized utility and compare it to what they were promised: if it is less then they react by punishing the leader for failing to deliver on the promise.

We now turn to the formal model. There is a symmetric game between two groups with a welfare restriction described in the next section. The groups are denoted by $k \in \{1, 2\}$, where each group has a representative follower. The followers choose actions $a_k \in \{C, F\}$, where C means cooperation and F means fight. Action profiles are denoted by $a \in A$, and all group k members receive utility $u_k(a_k, a_{-k})$ where $-k$ denote the other group. We assume that payoffs are distinct:

$$\text{for all } k, a \neq a' \text{ implies } u_k(a) \neq u_k(a'). \tag{1}$$

These utility functions give rise to the *underlying game*.

We now describe the leaders' game, which is built upon the underlying game. There are three leaders $\ell \in \{0, 1, 2\}$: two *group leaders* $\ell = 1, 2$ who have the same interest as group $k = \ell$, and a *common leader* $\ell = 0$ who cares about both groups. Denoting by $U^\ell(a)$ the utility leader ℓ obtains from profile a, we have $U^\ell(a) = u_\ell(a)$ for the group leaders $\ell = 1, 2$, and we take $U^0(a) = (u_1(a) + u_2(a))/2$ for the common leader $\ell = 0$ who thus shares the preferences of both groups.

As indicated, each leader presents his plan of action to their potential followers. The group leaders make offers only to their own group, the common leader to both groups. Specifically, a leader strategy is an $s^\ell \in A$, that is, an action profile in the underlying game. This represents an offer and a promise to the potential followers. The common leader presents his offer to both groups: each group is asked to play s^0_k and promised if they do so that the other group will play s^0_{-k}. Group leaders address only their own group: follow me and play s^ℓ_ℓ, the other group will play $s^\ell_{-\ell}$. The profile of leaders' strategies is $s \equiv (s^\ell)_{\ell \in \{0,1,2\}}$.

In addition to receiving direct utility the leaders may lose utility due to punishments by the followers; we now describe how. The followers in a group have the ability to impose a utility penalty P on their group leader and $P/2$ on the common leader. An interpretation is that the group leaders spend all their time with their group, while the common leader spends half his time with each group, and the extent of punishment is proportional to the amount of time the leader spends with that group. In the bulk of the paper we assume that the follower of group k considers the proposal of the corresponding group leader and the one by the common leader, but as a benchmark we also analyze the case in which followers ignore the proposal of the common leader, in which case there is no competition among leaders.[4]

Among the proposals they consider, the followers choose the one promising them the highest utility. That is, given a strategy profile s of the leaders, follower k chooses the proposal that maximizes $u_k(s^\ell)$ over the proposals they consider. The maximizer for group k is unique by assumption (1), though it may be proposed by more than one leader; denote it by $g^k(s) \in A$. Utility $u_k(g^k(s))$ is the utility promised to group k. Group k then implement their part in the chosen strategy, that is they play $g^k(s)_k$. Therefore, given a profile of leaders' strategies s, the *implemented action profile* will be $g(s) \equiv \left(g^k(s)_k\right)_{k=1,2} \in A$. This determines the utility of the groups, $u_k(g(s))$, and the direct utility of the leaders $U^\ell(g(s))$.

After actions are implemented and direct utility accrues, followers of group k impose a punishment to the followed leaders when the obtained utility is less than the one promised. Note that no counterfactual reasoning is required by the followers: they simply compare the promised utility to the actual utility. Precisely, if $u_k(g^k(s)) < u_k(g(s))$ then group k punishes $\ell \in \{0, k\}$ such that $s^\ell = g^k(s)$, where the punishment is P if $\ell = k$ and $P/2$ if $\ell = 0$.

[4] In this case the common leader may just not be there, or may be out of the game if the followers only consider proposals by leaders whom they can make accountable by punishment, and judge that the punishment they can inflict to the common leader is not substantial enough.

The sum of the direct utility and the punishments obtained as we have just described determine the payoff of leader ℓ, $V^\ell(s)$, for any strategy profile s. We let $\mathbf{1}\{\mathfrak{c}\} = 1$ if condition $\mathfrak{c}$ is true and zero otherwise. Then the payoff of a group leader $\ell = 1, 2$ is

$$V^\ell(s) = U^\ell(g(s)) - P \cdot \mathbf{1}\{\ell = k \;\&\; g^k(s) = s^\ell \;\&\; u_k(s^\ell) < u_k(g(s))\} \tag{2}$$

and of the common leader

$$V^0(s) = U^0(g(s)) - (P/2) \cdot \sum\nolimits_{k=1,2} \mathbf{1}\{g^k(s) = s^0 \;\&\; u_k(s^0) < u_k(g(s))\}. \tag{3}$$

We call the game played by the leaders $\ell \in \{0, 1, 2\}$, with $S^\ell = A$ and the utilities V^ℓ just defined, a *leaders game*. It is a finite game, hence an equilibrium in mixed strategies exists. We are interested in Nash equilibria in weakly undominated strategies of the leaders game. We call this a *leaders equilibrium*.

3. The Underlying Games

As indicated, we restrict attention to symmetric two-by-two underlying games with representative followers $k = 1, 2$. As indicated, each player has two possible actions, C and F. We assume that if both play C they get a higher von Neumann-Morgenstern utility than if they both play F.[5] Thus, denoting by u_k player k's utility

$$\text{for } k = 1, 2, \;\; u_k(C, C) > u_k(F, F). \tag{4}$$

Using invariance under monotonic linear transformations and (4), we set

$$u_k(C, C) = 1, u_k(F, F) = 0 \quad \text{for both } k. \tag{5}$$

Therefore, with $\lambda, \xi \in \mathbb{R}$, the family of games is the following (with $\lambda, \xi \notin \{0, 1\}$ by (1)):

	C	F
C	$1, 1$	ξ, λ
F	λ, ξ	$0, 0$

We are interested in the conditions on the political structure that, when cooperation is desirable, make it an equilibrium. Observe that in the leaders game each leader has four possible strategies (the action profile of the underlying game) so the leaders game is $4 \times 4 \times 4$.

Considering the combination of the two possible inequalities between λ and 1 on the one hand and ξ and 0 on the other, we have two sets of possible games. One has $\lambda > 1$, so the choice of F against C of the opponent is better than the choice of C: these are Prisoner's Dilemma if $\xi < 0$

[5]This is just a labeling convention: if two groups enjoy war more than peace, say in pursuit of honor in battle, then that is their way of cooperating and get higher utility.

and Chicken if $\xi > 0$. We call these *conflict games*, because (C, C) is not a Nash equilibrium of the game. The other set of possible games has $\lambda < 1$, so the choice of C against C of the opponent is better than the choice of F: they are Stag Hunt if $\xi < 0$ and Mutual Interest if $\xi > 0$. We call them *cooperation games*, because they admit (C, C) as an equilibrium.

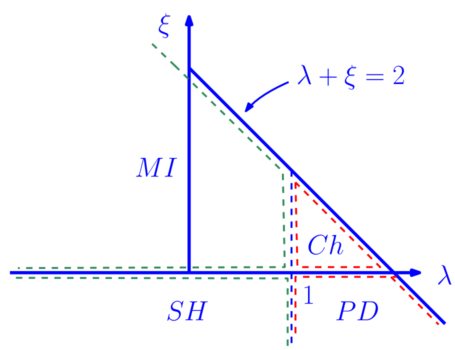
3.1. Welfare Restriction

The substantial restriction we introduce is that unilateral deviations from the best common action profile reduces average welfare, where we take simple average assuming that the groups have equal size:

$$\text{for both } k,\ u_k(C, C) > \frac{1}{2}u_1(F, C) + \frac{1}{2}u_2(F, C) \tag{6}$$

that is $\lambda + \xi < 2$. Together with $u_k(C, C) > u_k(F, F)$, this characterizes games where average players' payoff is highest at outcome CC. In this sense these are the games where conflict is detrimental. With this restriction the games we are studying can be visualized in (λ, ξ) space as in Figure 1, left panel.

Figure 1: **The family of games.** All payoffs are below the $\lambda + \xi = 2$ line. To the left of the $\lambda = 1$ line: above the horizontal axis ($\xi > 0$) there is Mutual Interest and below it is Stag Hunt. These are the cooperation games. To the right of $\lambda = 1$: above the axis we have Chicken, below it is Prisoners Dilemma. These are the conflict games.



3.2. Two Examples

We now consider two common examples of families of games that have already been considered in the literature. These examples, imposing a specific technology used to produce the utility values, carve out subsets of the space of the two parameters (λ, ξ). In this literature the analysis is usually much richer and complex than what may appear from the simple form we use here; considering these classic examples is useful however to put our approach in the perspective of a well known tradition.

1. Conflict over a Public Good

The first example is in the spirit of Esteban and Ray (2011) (see also Esteban and Ray (1994) and Esteban et al. (2012)), who focus on the issue of polarization. Consider a simple model of conflict between two large identical groups who compete for a public good, which is worth $v > 0$.

If both compromise each gets $v/2$. If one group compromises and the other fights the latter wins $(1/2+a)v-c$ and the loser is left with $(1/2-a)v$ where $0 \leq a \leq 1/2$ is the *degree of polarization; c* is the cost of fighting, which includes both direct costs of effort and monitoring costs associated with peer pressure and discouragement of free-riding. If both groups fight each has an equal probability of winning but there is also battle damage bc to each group where $b \geq 0$ may be interpreted as the *intensity of conflict*, so both get $v/2-(1+b)c$. After normalization this model results in the family of games defined by

$$\lambda = 1 + \frac{av/c - 1}{(1+b)} \quad \textit{and} \quad \xi = 1 - \frac{av/c}{(1+b)}.$$

Here $\lambda > 0$ and $\xi < 1$. Note that $\lambda + \xi = 2 - 1/(1+b)$, therefore the constraint (6) is satisfied for all values of the parameters.

The game will be one of Mutual Interest when the relative mobilization cost c/v is large and a is small so that $c/v > a$; in this case compromise is strictly dominant for each group ($\lambda < 1$ and $\xi > 0$). If $c/v < a$ but the intensity of conflict is large enough that $c/v \cdot (1+b) > a$ the game is one of Chicken ($\lambda > 1$ and $\xi > 0$). If both c/v and the intensity of conflict b are not too large relative to a so that $c/v < a$ and $(1+b) \cdot c/v < a$ the game is a Prisoners Dilemma (*PD*, with $\lambda > 1$ and $\xi < 0$).

In this setting $\lambda + \xi = 2 - 1/(1+b) > 1$, so in this setting Chicken and parts of PD and Mutual Interest are captured.

2. Strategic Complements versus Strategic Substitutes

Baliga and Sjöström (2020), see also Baliga et al. (2011), concentrate on strategic complements versus strategic substitutes. In Baliga et al. (2011), using our labels C and F, a player receives a payoff of 0 if they both cooperate, but $-d$ if he cooperates and the other fights. If a player fights, he pays a cost c for both actions of the other, but receives an additional utility μ if the other cooperates. Adding c to all entries and then dividing by c the utility matrix in is our general format, with

$$\lambda = \frac{\mu}{c} \quad \textit{and} \quad \xi = 1 - \frac{d}{c}. \tag{7}$$

They assume $\mu < d$, so (6) holds; indeed $\lambda + \xi < 1$ in this case. Also, if $\mu/c > 1$, then $d/c > 1$ and so $\mu/c > 1$ implies $\lambda > 1$ and $\xi < 0$. This is the Prisoners Dilemma game. On the other hand, if $\mu/c < 1$ (that is, $\lambda < 1$) then: if $d/c < 1$ then $\xi > 0$, which together with $\lambda < 1$ gives the Mutual Interest game; if $d/c > 1$ then $\xi < 0$, which together with $\lambda < 1$ gives the *Stag Hunt* game.

This case, since $\lambda + \xi < 1$, covers Stag Hunt together with parts of PD and Mutual Interest.

4. No Competition among Leaders

We start by studying the case where only group leaders are present, that is where $\ell \in \{1, 2\}$, and each group only considers proposals from their own group leader. There is no competition among leaders, follower k just plays what her group leader recommends. Our first result says that in this case the outcomes of the leaders game are the same as in the underlying game. This is actually true for any leaders game, with any number of groups, and even without the assumption (1).[6] Proving the statement for this more general case requires no additional effort, so we state it for this case:

Theorem 1. *For any leaders game, if each group only considers the proposal of their own group leader, then at the Nash equilibria of the leaders game the distributions of action profiles chosen by the groups are the same as those induced by the Nash equilibria of the corresponding underlying game.*

The proof is in Appendix. Thus, without competition among leaders there are no improvements over the outcomes of the underlying game.[7]

5. Games with Common Leader

We now introduce competition among leaders: followers consider the proposals of their group leader and of the common leader, so each group leader competes with the common leader. Having disposed of the no-competition case in the previous section we concentrate on this case in the sequel of the paper.

Notation. The "aggressive" proposal by group leader k "we play F and they play C" will be denoted by F^kC^{-k}. This is FC for leader 1 and CF for leader 2.

5.1. The Cooperation Games

We begin with the cooperation games (Mutual Interest and Stag Hunt). From Theorem 1 we know that in the game with only group leaders the equilibrium outcomes are those of the underlying game, so in the mutual interest game we have efficiency already without a common leader. The next theorem shows that with a common leader efficiency obtains also in Stag Hunt, for any value of P.

Theorem 2. *With a common leader, in the Mutual Interest and Stag Hunt games there is a unique leadership equilibrium for any value of P, with implemented action profile CC.*

Proof. The CC outcome is the most preferred by the common leader and she can guarantee that outcome by proposing it, because $u_k(F^kC^{-k}), u_k(F^kF^{-k}) < 1$ so the group leaders best response to CC by the common leader is to propose C to their group. □

5.2. The Conflict Games: Intuition and Summary

We use the Prisoner's Dilemma game to see how cooperation may arise in equilibrium when competition among leaders and accountability exist. Clearly, the proposal of "fighting on both sides" by both group leaders is beaten by the proposal of the common leader of cooperation of both

[6] The model trivially extends to the case of K groups: just take $k, \ell \in \{1, 2, \ldots, K\}$ instead of $k, \ell \in \{1, 2\}$.

[7] As the proof shows, the equilibrium strategy profile in the leaders game implementing a Nash equilibrium of the underlying game is not necessarily unique. But for any equilibrium in the leaders game the induced mixed action profile in the underlying game is unique.

groups, CC. This proposal is in turn easily beaten by the aggressive proposals F^kC^{-k} of the group leaders. However the two group leaders cannot both play F^kC^{-k} for sure, because they anticipate that these proposal would produce the bad outcome (with low utility for both groups, a utility they share) and the consequent punishment imposed by followers. The only equilibrium will be a mixed strategy one, in which group leaders randomize between aggressive play F^kC^{-k} and a conservative FF; and the common leader will mix too, between proposing cooperation CC and effectively opting out by playing FF. The probabilities at equilibrium of these various action profiles proposed by the leaders depend on the parameters, and vary across different equilibria. But when the cost of punishment is large group leaders will want to limit the probability of the aggressive proposals, which may imply costly punishment, thus leaving room for a winning cooperation proposal CC by the common leader.

Before going into details we summarize the payoff relevant properties of the leaders equilibria of the conflict games with a common leader as the punishment size becomes small or large.[8]

(1) As $P \to 0$ the limit equilibria replicate outcome distributions of equilibria of the corresponding underlying games.

(2) With the exception of the asymmetric pure equilibria in the Chicken game (see Theorem 4), as $P \to \infty$ the equilibrium probability of cooperation and average group payoff tend to 1.

So without punishment, the leaders mediation adds nothing to the underlying game. On the other hand competing, accountable leaders enable groups to achieve cooperation with surprisingly high probability in games where the group conflict is detrimental and the unmediated Nash equilibria are undesirable. This is the main message of the paper.

5.3. The Prisoners Dilemma

In this case the leaders game can be considerably simplified. For the group leaders, the strategies CC and C^kF^{-k} are weakly dominated by FF. For the common leader, the strategies CF and FC are then weakly dominated by FF for all $P > 0$. So the analysis is reduced to the game where the group leaders only play F^kC^{-k} or FF and the common leader plays only CC or FF. In summary, the game is reduced to a simpler game with three players, each player with two actions. This simplified game is presented in table 1.

The proof of the above statements is in the appendix, lemmas 9 and 10. Given this it can be shown that in equilibrium the common leader must play CC with positive probability, and in symmetric equilibrium the group leaders must play FC with positive probability.

The next theorem states what the equilibria of the leaders' game are. For P large, the implemented action profile converges to full cooperation. For small P, the equilibrium output is unique and both groups fight. The proof is in Appendix B.2.

[8] Assertion (1): for the chicken game this is a corollary to Theorem 21. For the prisoners dilemma this is part 1(a) of Theorem 11. Assertion (2): for the chicken game this is the last statement of Theorem 22. For the prisoners dilemma the claim follows from part 2(b) of Theorem 11 because if $P \to \infty$ then the probabilities $\tilde{q}$ and $\hat{q}$ there defined tend to 1 and $\tilde{p}$ and $\hat{p}$ (also defined in appendix) tend to zero.

Table 1: The game after elimination of weakly dominated strategies. The left panel shows utilities when the common leader plays CC; in the right panel are utilities when the common leader plays FF.

CC	CF	FF
FC	$0, -P, -P$	$\frac{\lambda+\xi-P}{2}, \lambda, \xi$
FF	$\frac{\lambda+\xi-P}{2}, \xi, \lambda$	$1, 1, 1$

FF	CF	FF
FC	$0, -P, -P$	$0, -P, 0$
FF	$0, 0, -P$	$0, 0, 0$

Theorem 3. *For P sufficiently low (more precisely $P < \min\{-\xi, \lambda + \xi\}$) there is a unique equilibrium outcome, in which both groups fight, and both groups get zero utility. For P sufficiently large (more precisely $P > \min\{-\xi, \lambda + \xi\}$) the equilibrium probability of the cooperation outcome converges to 1 as P becomes large.*

5.4. The Chicken Game

It follows from Theorem 1 that the pure equilibria of the underlying chicken game survive as leadership equilibria when there is no common leader. The presence of the common leader is not sufficient to change this fact:

Theorem 4. *The outcomes FC and CF of the underlying game are equilibrium outcomes of the leaders game for all (λ, ξ, P).*

This is proved in Appendix C, Lemma 19. But interesting new possibilities emerge in the mixed equilibrium we consider next.

Theorem 5. *There is a mixed strategy equilibrium of the leaders game in which the common leader plays CC with probability tending to 1, as P becomes large, and the group leaders play F^kC^{-k} with probability p and FF with probability $1-p$; the value of p converges to 0 as P becomes large. The equilibrium probability of cooperation converges to 1.*

This is proven in Theorem 22, which describes an equilibrium that exists for $P > \lambda + \xi$ in which the common leader randomizes between CC, FC and CF, with the probability of CC tending to 1 as P becomes large, and the group leaders randomize between F^kC^{-k} and FF, with the probability of F^kC^{-k} tending to zero as P grows large. In the limit the equilibrium implemented action profile is CC with probability 1.

In Theorem 21 it is shown that for $P < \lambda + \xi$ there is an equilibrium where the common leader plays CC for sure and group leaders randomize between F^kC^{-k} and FF. The probability of F^kC^{-k} in this equilibrium is

$$\tilde{p} = \frac{\lambda - 1}{P + \lambda + \xi - 1}.$$

This will be useful in Section 8, where we use the fact that in the more complex equilibrium of Theorem 22 the probability that the group leaders play F^kC^{-k} is smaller than $\tilde{p}$.

5.5. Comments on the Conflict Games

The two asymmetric equilibria outcomes in the underlying chicken game survive as leaders equilibrium implemented action profile for all P. It may come as a surprise that the inefficiency arising in the chicken game is harder to overcome than the prisoners dilemma. But the fact is that in the PD equilibrium of the underlying game both groups are worse off than in the cooperative outcome, and then a common leader may come to the rescue; in the chicken pure equilibria on the other hand one party is relatively well off (possibly better off than in the CC outcome), and when a group acts aggressively neither the opposite group leader nor a common leader can do anything to dissuade them.

As we know, at least in the Chicken game, better outcomes than in Nash equilibria may be reached also through the intervention of an external, uninterested mediator - in the correlated equilibria of the game. We compare leaders and correlated equilibria in Section 8. Of course fixing the (λ, ξ) parameters we already know what happens for $P \to \infty$. But as we shall see the mixed leaders equilibria fare better than the correlated equilibria of the underlying game already for moderate values of P. What makes the difference is that on the one hand the common leader is interested in cooperation (her most preferred outcome) and this is therefore what she tends to propose; and that on the other hand the group leaders are discouraged to make aggressive proposals by the threat of the punishment that may come as a consequence.

6. The Role of Differential Punishment

We have considered so far the hypothesis that the punishment for leaders is the same for group leaders and for common leaders. We now ask how outcomes would differ if the punishments were allowed to be different across leaders. This sheds some light on which leaders is more important to make accountable - and these turn out to be the group leaders. Specifically, we allow for different punishments for the group leaders and the common leader, letting P continuing to denote the common leader punishment and denoting by Q the common leader punishment, so that each group can punish the common leader by $Q/2$.

Theorem 6. *Assume that the conditions on P in Theorems 3 and 5, respectively $P \lesseqgtr \min\{-\xi, \lambda+\xi\}$ and $P \lesseqgtr \lambda + \xi$, hold for both P and Q. Then the structure of the mixed equilibria remains the same.*

To prove Theorem 6 only minor modifications are needed of the given arguments: in the various incentive constraints one has to specify which punishment is involved.[9]

6.1. Implications for the Prisoners Dilemma

We now examine the implications of Q in the Prisoners Dilemma. Roughly speaking we can think of smaller values of Q as applying to a common leader with a greater distance from the conflict.

[9] For the PD for example we only need to rewrite the preference conditions $CC \succcurlyeq_c FF$ and $FC \succcurlyeq_k FF$. The former was $(1-p)\,(1-p(1+P-(\lambda+\xi))) \geq 0$, and becomes $(1-p)\,(1-p(1+Q-(\lambda+\xi))) \geq 0$. And $FC \succcurlyeq_k FF$ was $P \leq q\,(P+\lambda-1-p(P+\lambda+\xi-1))$ and is now $P \leq q\,(P+\lambda-1-p(P+\lambda+\xi-1))$.

Specifically, take the case where $P > -\xi$ so that there is substantial punishment for the group leaders. If the common leader announces CC for sure then the group leaders are indifferent between FC and FF when both play FC with probability $\tilde{p}$. In this case the common leader gets 0 for announcing FF, while playing CC gives him $(1-\tilde{p})^2 + \tilde{p}(1-\tilde{p})\left[\lambda + \xi - Q\right]$, so inserting the value of $\tilde{p}$ we find that this is an equilibrium for

$$Q \leq \frac{P+\xi}{\lambda - 1} + \lambda + \xi,$$

that is, for Q not too large compared to P. For larger Q the equilibrium probability that the common leader plays CC is less than 1, but average group payoff still converges to 1 as P grows (this can be guessed from the fact that the probability $\tilde{p}$ with which the group leaders play aggressively goes to zero as P grows). This suggests that a situation is most favorable to cooperation when the group leaders have much to lose from failing to deliver and the common leader is relatively distant from the conflict, for then the latter surely puts forward his cooperation proposal, and this is likely to be accepted.

7. The Model, Civil Wars and Internal Politics

There are several predictions we can derive from the results presented in this paper. In this section we first summarize the predictions and then test them in recent history events.

First, the presence of common leaders makes cooperation certain in cooperation games and highly probable in conflict games if the group leaders can be substantially punished. Second, in the absence of a common leaders equilibrium outcomes of the leaders games are the same as those in the corresponding underlying games. Third, the level of feasible punishment of the common leader is not as important as the level possible in the case of group leaders: what matters is the presence of the common leader.[10]

In contexts where two groups confront each other with possibly conflicting interests, these findings advocate for the participation of an additional third party, called here a common leader, who benefits from a cooperation outcome. We analyze now some evidence concerning the frequency with which common leaders have emerged and the extent of their success in bringing cooperation, in the contexts of civil wars and internal politics of specific countries.

In civil wars within nations the obvious candidate to play the role of common leader is the United Nations (UN). Doyle and Sambanis (2000), Doyle and Sambanis (2006) conduct a detailed analysis of the UN operations since its onset, and interestingly reduce the typical instances of conflicts to

[10] In practice common leaders always have a reputation to maintain, but it is not always the case that the groups can effectively punish them directly. In the international context of civil wars in which the common leader is the United Nations the conflicting groups have usually little room for punishment. Different is the case, for example, when the US president Roosevelt intervened in the Russo-Japanese war, for in that context the intervenor had a strong interest in maintaining good relations with both parties.

two classes of games we analyze, the Stag Hunt and the Prisoners Dilemma.[11] Doyle and Sambanis (2006) analyze 121 civil wars between 1945 and 1999. Of these, 99 ended with a military victory or a truce - that is, without successful third parties interventions. Of the remaining 22, 14 ended with a negotiated settlement mediated by the UN, and in 12 of these cases there was no recurrence within 2 years from the settlement. Our model suggests that the conflicts ending in fights may be Prisoner Dilemmas with low punishments for the group leaders. In civil wars one may think that the punishment, seen as cost of failure, is particularly high (in the limit, death); but in war life is at risk whether you have promised victory or not, so that the additional punishment inflicted by followers is actually small. And in this case this is what the model predicts: high frequency of conflict, and sporadic occurrences of the cooperative outcome proposed by the common leader. The cases of recurrence after settlements should also be traced back to the nature of the underlying game: in a prisoners dilemma a cooperative outcome may appear, but it is fragile, as was the case in Rwanda for example. The cases of successful peacebuilding on the other hand, as also Doyle and Sambanis reckon, should be interpreted as Stag Hunt games, where the common leader drives the parties to the existing good equilibrium.

The UN has not been the only external player to play the role of common leader in international settlements. For example, the cooperative solution in the Brcko district (situated between Bosnia and Croatia), which had been left out of the Dayton Agreement due to the complexity of the situation, was brought about (in 1999, four years after Dayton) by an arbitral tribunal chaired by U.S. attorney Roberts Owen. In the path to the Paris Peace Accords of 1991, which brought an end to the long-standing conflict in Cambodia, the US and the Association of Southeast Asian Nations (ASEAN) played a key role.[12] In Mozambique the catholic Community of Sant'Egidio, firmly rooted in the territory, helped the UN's Secretary-General forge the peace agreement by offering negotiating space to the parties. In Sri Lanka the conflicting parties turned to Norway with the aim of reaching a cease-fire (this was signed in 2002 although hostilities started again in 2008). These are not recent developments: for an earlier example, in the Russo-Japanese war (1904-1905) the Treaty of Portsmouth was mediated by the US President Theodore Roosevelt, who was awarded the Nobel Prize in 1906 for his efforts.

In the case of internal politics we make the same predictions outlined earlier. To make these predictions operational we identify group leaders with parties, and common leaders with technical or neutral personalities who benefit from cooperative outcomes and suffer from failures mostly in terms of future reputation.

As a first example we take Dwight Eisenhower. The 1952 US Presidential election was a contest between two very distant group leaders.[13] Eisenhower was asked to be a candidate by both parties,

[11]Their focus is on the strategies that the UN should use. In the Stag Hunt case there is a good equilibrium so the intervenor's task is to help the parties coordinating towards that outcome. In prisoners dilemmas they suggest that external intervention should be directed to transforming the game, raising the costs of non-cooperation.

[12]For this and the next instances we mention see Hampson (2004).

[13]The Republican Robert A. Taft was an arch-conservative who opposed the New Deal, opposed US entry into World War II, opposed the Nuremberg trials, opposed NATO, the UN, and labor unions. The Democrat, Adlai

and although he ran as a Republican, he was close to the Democrat Harry Truman. Prior to the election he had no political affiliation, and a failed presidency would tarnish his reputation as a war hero. In office Eisenhower did act as a common leader, and his success at bringing the nation together is indicated by the fact that he has been the most popular president of the US post-war history.[14]

A renowned figure that may be regarded as common leader is Nelson Mandela. The apartheid system was introduced in 1948 in South Africa, and Mandela was jailed from 1964 to 1982 for opposing it. The group leaders, both on the white and black side, were radical. In his inaugural speech as President in 1994 he declared: "The time for the healing of the wounds has come. The moment to bridge the chasms that divide us has come."[15] His presidency gave a serious blow to the apartheid. And his government's results were positive on the economic side as well: per capita GDP fell at the rate of 1.35% per year in the decade preceding 1994, and rose by 1.4% per year in the following decade (data from FRED).

Another case in point is Italy, where common leaders emerged in two episodes of severe national emergencies, which we interpret as low-payoff outcomes resulting from mismanagement of the social and economic problems by the existing opposite parties. The first was in 1992-1994, which led to the governments of Ciampi and Dini (both coming from the Bank of Italy), and the second in 2011 when professor Monti from Bocconi University became prime minister. None of their governments included politicians. In 1992 industry and workers were strongly opposed over labor costs; judges Falcone and Borsellino were assassinated by the mafia in Sicily; the Italian Lira was devalued by 25-30% against Dollar and German Mark; and widespread corruption in political parties reached justice palaces and front pages of newspapers (the "Tangentopoli" scandal). Then in April 1993 abolition of public funding of political parties was voted in a referendum with a 92% consensus, and shortly after the incumbent government resigned and Ciampi was appointed prime minister. His government managed to take firms and labor unions to sign a historical agreement on the cost of labor, approved a reform of the electoral system, and started to put order in public finances to steer Italy into the eurozone. Ciampi served later as President of the Republic, and the Financial Times obituary article described him as "representing the finest tradition of public service".[16] In the elections following his resignation the first of the four Berlusconi governments moved in but lasted only six months, and after that Dini took over. He restarted where Ciampi had left, and approved the first substantial much needed pension reform. After that a few years of relative stability in Italian politics ensued. The second occurrence, in 2011, was in the midst of a financial crisis, where - with Berlusconi prime minister for the fourth time - the spread between interest rate on Italian debt and German bonds reached the alarming figure of 600 during the summer. The then president

Stevenson, along with Eleanor Roosevelt was the leader of the progressive movement: for labor unions and expanding the New Deal, and a leading internationalist who was instrumental in the founding of the UN. See Cotter (1983).

[14]See news.gallup.com/poll/116677/Presidential-Approval-Ratings-Gallup-Historical-Statistics-Trends.aspx

[15]Quotations from Mandela's Reuters obituary, taken from reuters.com/article/uk-mandela-obituary-idUKBRE9B417G20131206.

[16]https://www.ft.com/content/1e206c2c-7bfc-11e6-b837-eb4b4333ee43

Napolitano appointed Monti "in the interest of the common good" on November 16th.[17]. His first stability budget law included pension reform and real estate tax, and this reassured markets: in less than a month that value nearly halved. The underlying difficulties however remained, and the situation cooled down with the help of the "whatever it takes" of Mario Draghi, pronounced on the 26th of July, 2012.

8. Comparison with Correlated Equilibria

The leaders game built over an underlying game shares important features with the correlated equilibria of that underlying game: in both cases, thanks to a form of mediation, better outcomes than Nash equilibria can obtain; and in both solution concepts, leaders or the mediator suggest to followers an action profile, and followers respond. But the differences are deeper than the similarities.

In correlated equilibria the single mediator has no direct interest in the outcome; followers respond strategically to the action suggested privately to each, by updating the posterior on the action profile played by others, and would never want to punish the mediator. In the leaders game there are competing leaders with a direct interest in the outcome, so that their utility is affected by the action of the followers; the latter respond to the leaders' suggestions by choosing the best action profile from their point of view, and typically punish the chosen leaders with positive probability in equilibrium. Most importantly, although action profiles are implemented by the groups, the strategic interaction is among the leaders, not between the players of the underlying games.

Nonetheless both solution concepts produce sets of equilibrium action profiles, so the comparison from the point of view of welfare is of some interest. We take as measurement of welfare the average utility of players in the underlying game: so we sum the utility of the two groups and ignore the welfare of the leaders (which may include punishments). We call *efficient* the outcome where both players in the underlying game get a utility of one. In the leaders game we have an additional parameter to take into account, which is the punishment. Thus the comparison changes for different values of P. We are going to show that outcomes in the leaders game typically dominate correlated equilibria in average welfare. More precisely, we show that in all games the largest average utility at outcomes of equilibria of the leaders game is larger than the largest utility at correlated equilibrium outcomes.

The comparison is trivial in the case of the mutual interest game: both solution concepts predict a unique outcome, and the outcome is efficient. The comparison is also easy for the prisoners dilemma and the stag hunt game; but the two sets do not coincide, so the comparison is meaningful.

In the prisoner's dilemma, the outcome predicted by the leaders game is unique: it is the efficient outcome in the limit as P becomes large, and the zero utility outcome when $P = 0$. There is a unique correlated equilibrium of the underlying game, which is the zero utility outcome. Thus in this case the leaders' equilibrium dominates the correlated.

[17]https://presidenti.quirinale.it/Elementi/207444

In the stag hunt, the outcome of the leaders' game is unique, and it is the efficient outcome for any value of P. The set of correlated equilibria in not a singleton, so we consider the best and worst possible outcomes. The best outcome for correlated equilibria is the efficient one. Since the set of utilities in a correlated equilibria is convex, all the values between the best and worst utility are correlated equilibrium outcomes. The worst correlated equilibrium outcome is the one induced by the mixed strategies of the underlying game. Thus in this case too the leaders' equilibrium weakly dominates the correlated.

In the Chicken game the comparison is more complex, and we turn to it now. For fixed (λ, ξ) the comparison is straightforward:

Theorem 7. *In the Chicken game, given (λ, ξ), the average payoff in any correlated equilibrium of the underlying game is bounded away from 1, so for large enough P it is lower than the average payoff in the mixed equilibrium of the leaders game (which goes to 1 as $P \to \infty$, see Theorem 22).*

The proof of this is in Appendix D. There we also compare the worst equilirbia in the two concepts, and we prove that worst leaders equilibrium yields higher payoff than the worst correlated equilibrium.

Going back to the comparison between the best equilibria in the two concepts, if in the Chicken game we now remove the restriction of fixed (λ, ξ), and ask how large P must actually be for the mixed leaders equilibrium to beat the best correlated equilibrium of the underlying game. The result, which we state and prove in Appendix D, is the following.

Theorem 8. *For parameters in the interior of the chicken region, for values of P of the same order of magnitude as the players' payoffs, the leaders' equilibrium yields higher payoff than any correlated equilibrium of the underlying game.*

9. Conclusions

We have studied how political leadership can fundamentally alter outcomes in societies with group conflict. We rely on a model of leadership which may be useful in general environments: given an underlying game among players, we construct a game among leaders in which the leaders' strategies are action profiles proposed by each leader to the society of players-followers. Followers choose among the proposals to maximize their utility.

The main insight derived from our model and analysis is that conflict in polarized societies can be substantially reduced, *under appropriate conditions*, thanks to the mediation of interested leaders. The existence of leaders by itself cannot accomplish anything useful: the equilibrium outcomes are the same as in the game with no leaders. With common leaders, our analysis has identified two main forces: competition among leaders and accountability. If there is competition among leaders, then in general cooperation and good outcomes are possible when the accountability of leaders is sufficiently large. In the limit of high accountability, full cooperation is realized. Our results seem to temper the bleak picture that may emerge from the literature on group conflict reviewed in section

(1.1): a truce among groups in conflict is possible, under appropriate conditions. However, one has to put this conclusion in the appropriate perspective.

Our setup relies on simplifying assumptions, and some of these assumptions are in contrast with important real world regularities. In the model, leaders share precisely the utility of their constituencies, so their incentives are perfectly in line with those of the groups. Leaders do not have a political career to pursue, nor derive utility from being leaders. Leaders cannot profit directly or indirectly on their position. The common leader in particular is assumed to share the interests of society as a whole. Followers, on their part, make the task of the leaders as easy as possible: they hear what the leaders say, and take their promises at face value, with the understanding that punishment will follow if the leader does not deliver. Finally, punishment must be sufficiently high for cooperation to arise. Fortunately, our analysis makes clear the leaders' role, so it can be taken to provide the best case scenario for possible positive effects of mediation in group conflict. Systematic empirical research will have to decide which are the realistic ranges of the losses groups can impose on leaders.

The behavior of followers in our model is extremely simplified, but we do not consioder assumption of unsophisticated behavior completely unrealistic: in large and complex societies, understanding the structure of payoffs from social actions is at the same time very hard (because societies are complex) and unrewarding (because the action of each player - even when he has acquired enough information to evaluate the best choice - is in itself irrelevant). Thus a first simple approximation is to assume, as we do, that followers just consider the promised utility, and choose the highest.

A natural extension of the model presented here, to consider a more realistic behavior of followers, is a foundation of their behavior based on a model of information acquisition on relevant parameters affecting the utility of players. This information is hard to gather, so in our model it is delegated to leaders or parties, which can do that through costly effort, and then send messages (for example, political programs) to the entire society. Followers may then interpret the signals sent in the light of what they know and choose rationally the best action. In a different context, a similar idea is presented in Matějka and Tabellini (2021).

Appendix A. Proof of Theorem 1

The statement of the theorem is given here in the more general case in which there are $K \geq 2$ groups. The definition of the leaders' game is a natural extension of the one provided for the case $K = 2$.

Theorem (Theorem 1 in the text). *For any leadership game the outcomes in the underlying game induced by the the Nash equilibrium of the leadership game are the same induced by the Nash equilibria of the underlying game.*

Proof. For a mixed strategy $\hat{\sigma}^k$ of leader k we let $\hat{\sigma}^k_{A_k}$ the induced distribution on A_k. Our first claim is that

$$\forall \hat{\alpha} \in NE(UG) \exists \hat{\sigma} \in NE(LG) : \forall k,\ \hat{\sigma}^k_{A_k} = \hat{\alpha}_k, \tag{A.1}$$

where $NE(UG)$ and $NE(LG)$ denote the sets of Nash equilibria of the underlying game and leaders' game respectively. Consider a mixed action profile $\hat{\alpha} \in NE(UG)$. For any action $b_k \in \mathrm{supp}(\hat{\alpha}_k)$ choose

$$a_{-k}(b_k) \in \mathrm{argmin}_{c_{-k} \in A_{-k}} u_k(b_k, c_{-k}). \tag{A.2}$$

Define now $\hat{\sigma}^k$ as:

$$\hat{\sigma}^k(a) \equiv \sum\nolimits_{a_k \in A_k} \hat{\alpha}(a_k) \delta_{(a_k, a_{-k}(b_k))}(a). \tag{A.3}$$

If all leaders j different form k follow the strategy defined in (A.3) then leader k is facing the probability on A^{-k} given by $\hat{\alpha}_{-k}$. Consider now a possible strictly profitable deviation $\hat{\tau}^k$ from $\hat{\sigma}^k$. Since by following $\hat{\sigma}^k$ the k leader incurs no punishment cost, the increase in net utility to leader k from $\hat{\tau}^k$ is at least as large as the increase in direct utility, and the direct utility is the utility of the followers. Thus $\hat{\tau}^k$ would have a marginal on A_k that is a profitable deviation for player k from $\hat{\alpha}_k$ against $\hat{\alpha}_{-k}$, a contradiction with $\hat{\alpha} \in NE(UG)$.

The second claim is:

$$\forall \hat{\sigma} \in NE(LG),\ \text{if } \hat{\alpha}_k \equiv \hat{\sigma}^k_{A_k},\ \text{then } \hat{\alpha} \in NE(UG). \tag{A.4}$$

Consider in fact a strictly profitable deviation β_k from $\hat{\alpha}_k$ of a player k in the underlying game. Extend β_k to a profitable deviation τ^k in the leaders game of the k^{th} group leader following the construction in equations (A.2) and (A.3). This deviation would insure for group leader k, the same utility as β_k, which would then be higher than $\hat{\sigma}^k$, since the direct utility of τ^k is higher than $\hat{\sigma}^k$, and its punishment cost is zero; a contradiction with the assumption that $\hat{\sigma}^k$ is a best response. □

Appendix B. Analysis of the Prisoner's Dilemma

Appendix B.1. Elimination of Weakly Dominated Strategies

We begin with some preliminary Lemmas to eliminate weakly dominated strategies.

Lemma 9. *For group-k leader the strategies CC and $C^k F^{-k}$ are weakly dominated by FF.*

Proof. We let $k = 1$. Fix any profile s^{-k} of the other leaders.

Consider CF first. Suppose that $g(CF, s^{-k})_1 = F$; then group 1 must have accepted a proposal FF or FC by the common leader, so that by playing CF or FF group-1 leader gets the same payoff (λ or 0, no punishment). Suppose $g(CF, s^{-k})_1 = C$; then the common leader must have proposed CF as well and group-1 leader gets $\xi < 0$, while in this case by proposing FF she gets 0 and no punishment.

Now consider CC and suppose first $g(CC, s^{-k})_1 = F$; then group 1 must have accepted a proposal FC by the common leader, and therefore CC and FF yield the leader the same payoff. Suppose $g(CC, s^{-k})_1 = C$ so that her proposal is accepted; the competing offers may have been CC, CF or FF; if all other proposals are CC then her payoff does not change if she plays FF; if there is a CF or an FF by some $\ell \neq 1$ then group-1 leader is strictly better off by playing FF (she gets zero, while with CC she gets $\xi - P$). □

In view of this lemma we may assume that group leader k plays only F^kC^{-k} or FF; we let p_k denote the probability of F^kC^{-k}.

Lemma 10. *The probability that the common leader plays either CF or FC is zero.*

Proof. We do it for CF. This proposal is rejected by group 1 who will play F, and accepted for sure by group 2 who will play F and punish the common leader. She is better off by playing FF (strictly if $P > 0$). □

Appendix B.2. Nash Equilibria in Prisoners' Dilemma

In the previous section we have simplified the leaders' game when the underlying game is the prisoners' dilemma to a three players game, each player with two actions. This simplified game is reported in table 1 of the main text. Thanks to this simplification, we can describe a strategy profile of the three players with a vector of the form (q, p^1, p^2) where q is the probability that the common leader plays CC ($(1-q)$ that he plays FF), and p^k the probability that the k group leader plays FC ($(1-p^k)$ that he plays FF).

The next theorem characterizes the equilibria of the leaders' game when the underlying game is the Prisoners' Dilemma. We first introduce some notation. The pair $(\hat{q}, \hat{p})$ in (B.1) describes a pair of mixed strategies in the simplified game ($\hat{q}$ for the common leader and $\hat{p}$ for each of the group leaders). It does not give full cooperation, but the induced outcome converges to cooperation as P becomes large, because $\hat{q}$ converges to 1 and $\hat{p}$ converges to 0.

$$\hat{q} \equiv \frac{P}{P + \lambda - 1 - \frac{P+\lambda+\xi-1}{P-(\lambda+\xi-1)}}\,, \qquad \hat{p} \equiv \frac{1}{P + 1 - \lambda - \xi} \tag{B.1}$$

The equation (B.2) defines a different pair of mixed strategies (actually pure for the common leader); note that $\tilde{p}$ converges to 0 as P becomes large.

$$\tilde{q} = 1\,, \qquad \tilde{p} \equiv \frac{\lambda - 1}{\lambda - 1 + P + \xi} \tag{B.2}$$

Finally, the inequality (B.3) links the three parameters together, and decides (see the last point in theorem 11) whether the equilibrium as P becomes large is B.1 or B.2.

$$\xi + (\lambda - 1)(\lambda + \xi) > (\lambda - 2)P \tag{B.3}$$

We can now present the theorem:

Theorem 11. *In the leaders' game with prisoners' dilemma underlying game:*

1. *If $P < \lambda + \xi$:*
   (a) *If $P < -\xi$ the equilibria are all $(q, 1, 1)$ for any $q \in \left(\frac{P}{-\xi}, 1\right]$;*
   (b) *If $P > -\xi$ the equilibria are $(1, \tilde{p}, \tilde{p})$ and the set $\{(1, 1, 0), (1, 0, 1)\}$*
2. *If $P > \lambda + \xi$:*
   (a) *If $P < -\xi$ the equilibria are all $(q, 1, 1)$ for any $q \in \left(\frac{P}{-\xi}, 1\right]$, and the set*

$$\{(q, \hat{p}, \hat{p}) : \min\{-\frac{P}{\xi}, \frac{P}{P + \lambda - 1}\} < q < \max\{-\frac{P}{\xi}, \frac{P}{P + \lambda - 1}\}\};$$

   (b) *If $P > -\xi$:*
      i. *if the inequality (B.3) holds, then the equilibria are $(1, \tilde{p}, \tilde{p})$;*
      ii. *if the inequality (B.3) does not hold, then the equilibria are $(\hat{q}, \hat{p}, \hat{p})$;*

Proof. The proof follows from consideration of the cases examined below in Appendix B.3. □

We turn to the proof of theorem 3:

Proof. The proof follows from theorem 11. As P becomes small, the only relevant case is 1.(a), in which both $P < -\xi$ and $P < \lambda + \xi$. In this case the two group leaders play FC and CF respectively for sure, so the outcome in the underlying game is (F, F) for sure.

As P becomes large, the only relevant case is 2.(b), in which both $P > \lambda + \xi$ and $P > -\xi$. In this case the nature of the equilibrium is decided by the inequality B.3. Note that whether this equality holds or not for large P depends on whether λ is smaller or larger than 2. □

Appendix B.3. Analysis of Equilibria in PD

We will identify all the equilibria in the game; the analysis is organized considering three possible cases for the value of q, namely $q = 0$, $q = 1$ and then $q \in (0, 1)$. We concentrate on the interesting cases in which the relevant inequalities among combinations of parameters hold strictly.

Equilibria with $q = 0$

Lemma 12. *If $P > 0$, there is no equilibrium with $q = 0$*

Proof. If the common leader sets $q = 0$ then the leaders' game is the bottom panel of table 1 (ignoring the common leader's utility). This game has a unique Nash Equilibrium in dominant strategies in which both group leaders play FF. At this profile of actions of group leaders, CC yields 1, and FF yields 0, to the common leader, hence setting $q = 1$ is the best response. □

Equilibria with $q = 1$

In the first lemma we deal with the case of small P:

Lemma 13. *If $\xi < -P$ then there is a unique equilibrium with $q = 1$, with $(q, p_1, p_2) = (1, 1, 1)$.*

Proof. Since $\lambda > 1$ and $\xi < -P$, if $q = 1$ we see from table 1 that the action FC is dominant for the first group leader CF for the second). When group leaders play the action profile (FC, CF) then both CC and FF give utility 0 to the common leader, hence $(1, 1, 1)$ is the only equilibrium with $q = 1$. □

Lemma 14. *If $\xi > -P$:*

1. *There are two equilibria where group leaders play pure strategies: $(q, p_1, p_2) \in \{(1, 0, 1), (1, 1, 0)\}$ if and only if $\lambda + \xi - P > 0$.*
2. *There is an equilibrium where group leaders play a mixed strategy if and only if:*

$$\xi + (\lambda - 1)(\lambda + \xi) + (2 - \lambda)P > 0. \tag{B.4}$$

The mixed strategy is $\tilde{p}$ in equation (B.5).

Note that, for fixed λ and ξ as P becomes large the equilibria as in lemma 14 fail to exist, and also equilibria as in case (1) of lemma 13 fail to exist, and the same for the equilibria in case (2) of the same lemma when $\lambda > 2$. In summary equilibria with $q = 1$ exist for P large if and only if $\lambda < 2$.

Proof. If $\xi > -P$ then at $q = 1$ the game among group leaders has three equilibria, the two pure strategies (FF, CF), (FC, FF) and a mixed one with:

$$p^1 = p^2 = \frac{\lambda - 1}{\lambda - 1 + P + \xi} \equiv \tilde{p} \tag{B.5}$$

Note that $\lambda > 1$ and our assumption that $\xi > -P$ insure that $\tilde{p} \in (0, 1)$.

We first consider the possible equilibria where group leaders play pure strategies:

1. If $\lambda + \xi - P > 0$ then there are two equilibria, $(q, p_1, p_2) = (1, 0, 1), (1, 1, 0)$. This follows because CC gives $\frac{\lambda+\xi-P}{2}$, while FF gives 0 to the common leader.
2. If $\lambda + \xi - P < 0$ then there are no equilibria $(1, p_1, p_2)$ with $p_i \in \{0, 1\}$, because in this case the utility to the common leader from CC is lower than the one from FF.

We then consider the the possible equilibria where group leaders play a mixed strategy. At any mixed strategy profile (p, p), with $p \in (0, 1)$ of the group leaders the common leader gets

$$(1 - p)^2 + 2p(1 - p)\frac{\lambda + \xi - P}{2}$$

which is larger than 0 (hence CC better than FF) if and only if (B.4) holds. □

Equilibria with $q \in (0, 1)$

To set up the analysis we assume that the common leader is playing q and compare the corresponding expected payoff from FC and FF for group leader in the two cases: group leader plays CF and FF (thus, four comparisons overall). In the first case FC is better than CC if and only if

$$q > -P/\xi \tag{B.6}$$

In the second case FC is better than CC if and only if

$$q > \frac{P}{P + \lambda - 1} \tag{B.7}$$

In lemmas 15 and 16 we consider the two extreme possible cases for q:

Lemma 15. *There is no equilibrium with* $0 < q < \min\{-\frac{P}{\xi}, \frac{P}{P+\lambda-1}\}$.

Proof. The condition on q implies that the action FF is dominant for both group leaders, and so for any such q the payoff to the common leader at the best response of the group leaders from CC is 1, and from FF is zero, so no $q \in (0, 1)$ can be part of an equilibrium. □

Lemma 16. *There is an equilibrium with any q such that* $\max\{-\frac{P}{\xi}, \frac{P}{P+\lambda-1}\} < q < 1$*, of the form* $(q, 1, 1)$.

Of course the set of such q's may be empty; this is the case when P is large.

Proof. The condition on q implies that FC for group leader 1 (CF for 2) is dominant. At this best response (FC, CF) of the group leaders, both CC and FF give a payoff of 0, hence any q (in particular any satisfying that condition) is part of an equilibrium of the form described. □

Next we consider the intermediate cases for the values of q. At these values of q the game with $q$expected payoffs of group leaders has three equilibria, two pure strategies and one mixed. We deal with pure strategies in lemma 17.

Lemma 17.
1. $\frac{P}{P+\lambda-1} < q < -\frac{P}{\xi}$ *then there is no equilibrium with* $p_i \in \{0, 1\}$ *(that is, with group leaders playing pure strategies)*
2. *For any value* $-\frac{P}{\xi} < q < \frac{P}{P+\lambda-1}$*, there is an equilibrium in pure strategies for group leaders of the form* $(q, 1, 1)$.

Proof. For the first case, consider for example the profile (FF, CF) (the other is (FC, FF)). In this case CC gives $\frac{\lambda+\xi-P}{2}$, and FF gives 0. Considering only the cases in which the inequalities holds strictly, it follows that the best response of the common leader to this strategy profile of the group leaders is either $q = 0$ or $q = 1$, hence not in the open interval $(0, 1)$.

For the second case, note that with value of q in that range with the strategy profile (FC, CF), both CC and FF give zero to the common leader, hence $(q, 1, 1)$ with any q in the range is an equilibrium. Instead, the other possible equilibrium with q-expected payoffs has CC giving value 1 to the common leader, and FF giving 0, hence no equilibrium with the q component in the open interval $(0,1)$ can exist. $\square$

Lemma 18. *An equilibrium with* $\min\{-\frac{P}{\xi}, \frac{P}{P+\lambda-1}\} < q \le \min\{\max\{-\frac{P}{\xi}, \frac{P}{P+\lambda-1}\}, 1\}$ *exists, with a mixed strategy* $(\hat{q}, \hat{p}, \hat{p})$ *defined in equations B.8 and B.10 below.*

Proof. For q to be part of an equilibrium, the common leader has to be indifferent between CC and FF which is true if and only if:

$$p = \frac{1}{P+1-\lambda-\xi} \equiv \hat{p} \tag{B.8}$$

The indifference for group leader 1 (for example) between FC and FF requires:

$$-pP + (1-p)(q\lambda - (1-q)P) = pq\xi + (1-p)q$$

which is rewritten as:

$$p = \frac{P+\lambda-1-P/q}{P+\lambda+\xi-1} \equiv f(q) \tag{B.9}$$

Combining equations B.8 and B.9 we conclude that an equilibrium with q in the range exists if both $f(q) = \hat{p}$ and

$$\min\{-\frac{P}{\xi}, \frac{P}{P+\lambda-1}\} < q < \max\{-\frac{P}{\xi}, \frac{P}{P+\lambda-1}\}.$$

and since $f(-\frac{P}{\xi}) = 1$ and $f(\frac{P}{P+\lambda-1}) = 0$ with f strictly increasing, there is unique $\hat{q}$ in the given range such that

$$f(\hat{q}) = \hat{p}. \tag{B.10}$$

it is easy to check that this $\hat{q}$ is indeed the value in equation (B.1).

Note that for P large, $\max\{-\frac{P}{\xi}, \frac{P}{P+\lambda-1}\} = -\frac{P}{\xi} > 1$, hence in this case we must check whether an equilibrium exists with $\frac{P}{P+\lambda-1} < q < 1$. Since $f(\frac{P}{P+\lambda-1}) = 0$, we now compare compare $f(1) = \frac{\lambda-1}{P+\lambda+\xi-1}$ and $\hat{p}$; so a solution exists if and only if $\frac{\lambda-1}{P+\lambda+\xi-1} > \frac{1}{P+1-\lambda-\xi}$; this in turn is equivalent to:

$$(\lambda-2)\,P > \lambda(\xi+\lambda-1) \tag{B.11}$$

.

So if $\lambda > 2$ we have an interior equilibrium for large values of P. In the other case (that is, $\lambda < 2$) we have $f(q) < \hat{p}$ for all $\frac{P}{P+\lambda-1} \le q \le 1$, and the equilibrium is $(1, \tilde{p}, \tilde{p})$ with $\tilde{p}$ introduced earlier in equation (B.5). $\square$

Appendix C. Analysis of the Chicken Game

Appendix C.1. The Pure Strategy equilibria

We describe here symmetric equilibria, so we formulate the lemma focusing on one outcome, (F,C). The same statement holds for the outcome (C,F).

Lemma 19. *In the Chicken game the outcome (F, C) of the underlying game is an equilibrium outcome of the leaders' game for all (λ, ξ, P).*

Proof. We write $BR^{\ell}(a^0, a^1, a^2)$ the best response of leader ℓ to the profile (a^0, a^1, a^2). The proof has three parts, for $\ell \in \{0, 1, 2\}$:

$$FC \in BR^{\ell}(FC, FC, FC) \tag{C.1}$$

So in each step we proceed from the assumption that the other leaders are playing (F, C) and consider the best response of the leader under consideration. We then examine the expected utility from the different possible choices of the leader under consideration, and claim that conclude that his best response is (F, C).

Consider first $\ell = 2$. Given $a^1 = a^0 = (F, C)$, group 1 will choose F no matter what the other leader offers, because this is the largest utility it can receive, and group 2 has the proposal (F, C) of the common leader. Considering the possible choices of a^2: (C, C) gives a utility $\xi - P$ (because $\xi < 1$, so group 2 will follow leader 2 , but the outcome then will be (λ, ξ) rater than the implicit promise $(1, 1)$ of leader 2, and hence leader 2 will be punished. (C, F) gives a utility $-P$ (group 2 will follow leader 2 and play F but the outcome is then $(0, 0)$ and so leader 2 gets the 0 utility and the punishment because the realized 0 is smaller than the promised λ). (F, C) gives a utility ξ (because both common leader and leader 2 promise the same utility profile). Finally, (F, F) gives a utility ξ (because the associated utility vector is $(0, 0)$, and common leader is promising ξ). Our claim follows.

Consider next $\ell = 0$. We proceed noting that $a^1 = a^2 = (F, C)$, and thus group 1 is choosing F. (C, C) gives a utility of $\frac{\lambda+\xi-P}{2}$ (because group 1 will choose F, following the group leader, while group 2 will choose C, following the common leader, expecting utility 1 rather than the ξ proposed by the group leader. Thus the outcome is (F, C), thus the common leader direct gets utility $\frac{\lambda+\xi}{2}$, and group 2 punishing the common leader). (C, F) gives a utility of $-P/2$ (because group 1 will follow leader 1, and group 2 will follow the common leader and play F expecting λ. Thus the outcome is (F, F) and average utility of groups equal to 0 and punishment of common leader by group 2. (F, C) gives a utility of $\frac{\lambda+\xi}{2}$ (because all leaders are proposing the same action profile). (F, F) gives a utility of $\frac{\lambda+\xi}{2}$ (because the proposal of the common leader will be ignored).

Consider finally $\ell = 1$. Assuming $a^0 = a^2 = (F, C)$, we note that group 1 is considering the utility λ from the common leader (with choice C), and group 2 is considering the utility ξ from both common leader and group leader 2. Group 1 is choosing F, following the common leader, no matter what group leader 1 is going to propose. The choice $a^1 = (F, C)$ gives leader 1 a utility of λ (group 1 is choosing F, because this is then the only proposal they receive, and group 2 is choosing C); but λ is the largest possible utility, hence (F, C) is a best response of group leader 1. $\square$

The next lemma shows that different degrees of communications between groups and leaders does not alter this conclusion:

Lemma 20. *In the Chicken game the outcome (F, C) of the underlying game is an equilibrium outcome of the leaders' game for all (λ, ξ, P) and for all γ functions.*

Proof. We adopt the formulation in which all leaders formulate a proposal, and the γ function only selects a special subset of the proposal that each group observes. So there is in each case a vector (s^0, s^1, s^2) of proposals, one from each leader. In the case $\gamma^1 = \{0,1\}$, group 1 observes (s^0, s^1), whereas in $\gamma^1 = \{0,1,2\}$ group 1 observes (s^0, s^1, s^2), and so on.

Consider the pure strategy equilibrium (FC, FC, FC) identified, in the case $\gamma(1) = \{0,1\}$, in lemma (19). Consider now the same pure strategy profile, but in the game where $\gamma(1) = \{0,1,2\}$, and consider the best response of leader 2. We claim that his best response in the same in the games with the two different γ functions. In fact, the expected utility from the choice of each strategy profile leader 2 can take is the same in both games, since the other two leaders in the pure strategy profile are taking the same strategy. Hence the strategy profile (FC, FC, FC) is an equilibrium irrespective of the γ function. □

Theorem 21. *For $P \leq \lambda + \xi$ or $2\lambda + \xi \leq 3$ there is an equilibrium where the common leader plays CC for sure, and the group leaders play $F^k C^{-k}$ with probability $\tilde{p}$ and FF with probability $1 - \tilde{p}$, with $\tilde{p}$ as defined in B.2.*

Proof. The utility matrix when the common leader plays CC is the following:

	CC	FC	CF	FF
CC	$1,1,1$	$1,1,1$	$\frac{\lambda+\xi-P}{2}, \xi - P, \lambda$	$1,1,1$
CF	$1,1,1$	$1,1,1$	$\frac{\lambda+\xi-P}{2}, \xi, \lambda$	$1,1,1$
FC	$\frac{\lambda+\xi-P}{2}, \lambda, \xi - P$	$\frac{\lambda+\xi-P}{2}, \lambda, \xi$	$0, -P, -P$	$\frac{\lambda+\xi-P}{2}, \lambda, \xi$
FF	$1,1,1$	$1,1,1$	$\frac{\lambda+\xi-P}{2}, \xi, \lambda$	$1,1,1$

Consider first a group leader, given the others' strategies: if she plays FF he gets

$$p\xi + 1 - p = 1 - p(1 - \xi)$$

while if she plays FC she gets

$$-pP + (1-p)\lambda = \lambda - p(\lambda + P)$$

so indifference between FF and FC holds if and only if:

$$p = \frac{\lambda - 1}{\lambda - 1 + \xi + P} = \tilde{p}.$$

This is smaller than 1 because $\xi > 0$. For a group leader proposing C cannot improve utility, since C is proposed by the common leader already. And indeed as we see from the utility matrix CF yields the same utility as FF and CC is weakly worse.

Consider now the common leader. The reduced utility matrix when she plays CC is this

	CF	FF
FC	$0, -P, -P$	$\frac{\lambda+\xi-P}{2}, \lambda, \xi$
FF	$\frac{\lambda+\xi-P}{2}, \xi, \lambda$	$1,1,1$

so by playing CC she gets

$$(1-p)\left(1+p(\lambda-1+\xi-P)\right).$$

This value is strictly positive because it is easily verified that at $p=\tilde{p}$ one has $1+p(\lambda-1+\xi-P)>0$. From the reduced utility matrix in the case in which the common leader plays FF:

	CF	FF
FC	$0,-P,-P$	$0,-P,0$
FF	$0,0,-P$	$0,0,0$

we see that FF gives zero, less than CC.

Consider lastly the utility from playing FC. The utility matrix is

	CF	FF
FC	$-P/2,-P,-P$	$\frac{\lambda+\xi}{2},\lambda,\xi$
FF	$-P/2,0,-P$	$\frac{\lambda+\xi}{2},\lambda,\xi$

so her utility is

$$p^2(-P/2)-p(1-p)(P-(\lambda+\xi))/2+(1-p)^2(\lambda+\xi)/2$$

Thus the common leader prefers CC to FC if the following difference is positive:

$$p^2P+p(1-p)\left((\lambda+\xi)-P\right)+(1-p)^2(2-(\lambda+\xi))$$

so for $P\leq\lambda+\xi$ this is certainly positive for any (ξ,λ) pair in the chicken region. Consider next $P>\lambda+\xi$. As $P\to\infty$, since $\tilde{p}\to 0$ and $\tilde{p}P\to\lambda-1$ the limit of the above difference is easily computed to be $(1/2)\left(3-2\lambda-\xi\right)$, which is positive for $2\lambda+\xi\leq 3$. We now show that for $2\lambda+\xi\leq 3$ the above difference is strictly positive for all $P>\lambda+\xi$. Neglecting the $1/2$ factor we can re-write it as

$$2p^2\left(1+P-(\lambda+\xi)\right)-p(P+4-3(\lambda+\xi))+2-(\lambda+\xi).$$

We are assuming $P>\lambda+\xi$ so the first term is positive; and we now show that the remaining part is positive as well, which inserting $\tilde{p}$ becomes

$$[2-(\lambda+\xi)]\left[\lambda-1+\xi+P\right]>(\lambda-1)\left(P+4-3(\lambda+\xi)\right).$$

This is found to be equivalent to

$$P\left(3-2\lambda-\xi\right)>2(\lambda-\xi-1)-(\lambda+\xi)(2\lambda-\xi-2)$$

so since $3-2\lambda-\xi$ it suffices to show that the right member is negative, equivalently $(\lambda+\xi)(2\lambda-\xi-2)>2(\lambda-\xi-1)$; this in turn can be checked to simplify to $2(\lambda-1)^2>\xi(\xi-\lambda)$ which is true since $\xi<1<\lambda$ implies $\xi-\lambda<0$. □

It may be useful to state the following

Corollary. *For $P=0$ the outcome distribution of the above equilibrium is the same as in the mixed equilibrium of the underlying game.*

Proof. For $P=0$ we have $\tilde{p}=p(F)$ where $p(F)$ is the probability of F in the mixed equilibrium of the underlying game. Then the claim follows because in the leaders equilibrium: the probability of FF is $\tilde{p}^2$; outcomes FC and FF have probability $\tilde{p}(1-\tilde{p})$; and CC has probability $(1-\tilde{p})^2$. Given $\tilde{p}=p(F)$ this is as in the mixed equilibrium of the underlying game. □

We next state and prove the result concerning equilibrium in the case $P>\lambda+\xi$ and $2\lambda+\xi>3$. Recall that the difference utility from CC minus utility from FC is

$$(1/2)\left[p^2P-p(1-p)\left(P-(\lambda+\xi)\right)+(1-p)^2(2-(\lambda+\xi))\right]$$

We re-write this as

$$(P+1-(\lambda+\xi))\,p^2-\left[P/2-1+(3/2)\,(2-(\lambda+\xi))\right]p+(2-(\lambda+\xi))/2 \tag{C.2}$$

Theorem 22. *For each pair (ξ,λ) with $2\lambda+\xi>3$ there is a $\overline{P}(\xi,\lambda)>\lambda+\xi$ such that for $P\le\overline{P}$ the equilibrium in the previous theorem still exists. For $2\lambda+\xi>3$ and $P>\overline{P}$ the mixed leadership equilibrium can be described as follows. There is a $p(P)$, $0<p(P)<\tilde{p}$ such that the group leaders play FC with probability $p(P)$ and FF with probability $1-p(P)$; the common leader plays CC with probability q and FC and CF with probability $(1-q)/2$ each, with (writing p for $p(P)$)*

$$q=\frac{\xi+(1+p)P}{2\lambda+\xi-2-2p\,(\lambda+\xi-1)+P(1-p)}<1.$$

As $P\to\infty$ we have $p(P)\to 0$ and $q\to 1$.

Proof. It is clear from the proof of the previous theorem that for each pair (ξ,λ) with $2\lambda+\xi>3$ there is a $\overline{P}(\xi,\lambda)>\lambda+\xi$ such that for $P\le\overline{P}$ that equilibrium still exists (because for $P\le\lambda+\xi$ it is positive for any (ξ,λ) pair). Precisely, $\overline{P}(\Gamma)$ is the value at which for $p=\tilde{p}$ the function in (C.2) as a function of P is zero. Note that in this function, for fixed $P>\lambda+\xi$ the coefficient of p^2 is positive; the function is positive at $p=0$, and the derivative there is

$$\begin{aligned}&2p(P+1-(\lambda+\xi))-\left[P/2-1+(3/2)\,(2-(\lambda+\xi))\right]\big|_{p=0}\\&=-\,(3/2)\,(2-(\lambda+\xi))-P/2+1=-(1/2)\,[3\,(2-(\lambda+\xi))+P-2]<0\end{aligned}$$

because $P>\lambda+\xi$ whence

$$3\,(2-(\lambda+\xi))+P-2>3\,(2-(\lambda+\xi))+(\lambda+\xi)-2=2\,(2-(\lambda+\xi))\,.$$

At $p=1$ the value is $P/2>0$ so both roots are less than 1 (incidentally, the smaller one becomes smaller as P grows larger, in fact it goes to zero). For each $P > \overline{P}(\Gamma)$ the function is negative at $p = \tilde{p}$ (by construction). Define $p(P)$ to be the root of (C.2) on the left of $\tilde{p}$; so $0 < p(P) < \tilde{p} < 1$. By construction for $p = p(P)$ we have $CC \sim_c FC \sim_c CF$.

We let p to be the $p(P)$ defined above. Consider a group leader. If she plays FF she gets (in square brackets what the common leader plays)

$$q\,(1 - p(1-\xi)) + (1-q)/2\,(\lambda + \xi - p\lambda)$$

while by playing FC she gets: (note that $q + (1-q)/2 = (1+q)/2$)

$$((1+q)/2)\,[\lambda - p(\lambda + P)] - ((1-q)/2)P$$

so she is indifferent if

$$((1+q)/2)\,[\lambda - p(\lambda + P)] - (1-q)/2)P.$$

This simplifies to

$$q = \frac{\xi + (1+p)P}{2\lambda + \xi - 2 - 2p\,(\lambda + \xi - 1) + P(1-p)},$$

and it can be checked that $q < 1$ if and only if $p < \tilde{p}$, which is true by construction. This ends the equilibrium argument, since in this case it is apparent that no leader has a profitable deviation.

Finally, as $P \to \infty$ we have $p(P) \to 0$ since $p(P) < \tilde{p}$ and $\tilde{p} \to 0$; and given this it is immediate that $q \to 1$. □

Appendix D. Proof of statements in Section 8

We consider first the case of the best correlated equilibrium for fixed (λ, ξ). The incentive compatibility constraints in the definition of correlated equilibria have the value $\mu(FF)$ appearing in the two inequalities $\mu(a)(\lambda - 1) \geq \mu(FF)\xi$, with $a \in \{FC, CF\}$ (these are the inequalities corresponding to the communication of the action F). On the other hand, the value $\mu(FF)$ does not appear in the total welfare sum; thus in any solution $\hat{\mu}$ of the maximization of total welfare over the set of correlated strategies, necessarily $\hat{\mu}(FF) = 0$, that is:

$$\hat{\mu}(CC) + \hat{\mu}(CF) + \hat{\mu}(FC) = 1. \tag{D.1}$$

Adding the incentive compatibility constraint of the first and second player upon communication of the C action we obtain:

$$(\hat{\mu}(FC) + \hat{\mu}(CF))\xi \geq 2\hat{\mu}(CC)(\lambda - 1) \tag{D.2}$$

From (D.1) and (D.2) we conclude that the total probability on the two non cooperation action profiles FC and CF is bounded below:

$$\hat{\mu}(FC) + \hat{\mu}(CF) \geq \frac{2(\lambda - 1)}{2(\lambda - 1) + \xi} \tag{D.3}$$

In summary, the best correlated equilibrium is:

$$\hat{\mu}(CC) = \frac{\xi}{\xi + 2(\lambda - 1)}, \hat{\mu}(FC) = \hat{\mu}(CF) = \frac{\lambda - 1}{\xi + 2(\lambda - 1)}, \hat{\mu}(FF) = 0 \tag{D.4}$$

with average utility:

$$\frac{\xi + (\lambda + \xi)(\lambda - 1)}{\xi + 2(\lambda - 1)} \tag{D.5}$$

Since $\lambda + \xi < 2$ this is clearly less than 1.

We next turn to "worst against worst" comparison. We first compute the lowest correlated equilibrium payoff. For small P the Chicken game has a mixed leaders equilibrium whose payoff is increasing in P, so the lowest occurs for $P = 0$ where its outcome distribution is the same as in the mixed equilibrium of the underlying game. The asymmetric outcomes of the underlying game are leaders equilibrium outcomes as well, so the worst leaders equilibrium is either the mixed or one of the pure equilibria of the underlying game (whichever is worse). It is proved below that both yield higher payoff than the worst correlated equilibrium. We turn to details.

An argument analogous to the one above can be applied to determine the worst correlated equilibrium, $\underline{\mu}$, which is:

$$\underline{\mu}(CC) = 0, \underline{\mu}(FC) = \underline{\mu}(CF) = \frac{\xi}{2\xi + \lambda - 1}, \underline{\mu}(FF) = \frac{\lambda - 1}{2\xi + \lambda - 1} \tag{D.6}$$

with average utility:

$$\frac{(\lambda + \xi)\xi}{2\xi + \lambda - 1} \tag{D.7}$$

The mixed equilibrium in the underlying game is seen to yield payoff

$$\frac{\lambda\xi}{\lambda - 1 + \xi}$$

while the asymmetric pure equilibria give of course $(\lambda + \xi)/2$. Either of the two may yield lowest payoff, but both are easily verified to be higher than in the worst correlated equilibrium. Indeed: the mixed equilibrium is better than the worst CE if $\frac{\lambda\xi}{\lambda-1+\xi} > \frac{\xi(\lambda+\xi)}{\lambda-1+2\xi}$, that is if $1 > \xi$. and the asymmetric beats it if $(\lambda + \xi)(\lambda - 1) > 0$. This proves the claim in the text.

For the sake of completeness we compare mixed and asymmetric equilibria of the underlying chicken game. Asymmetric better than mixed if

$$\frac{\lambda\xi}{\lambda - 1 + \xi} < \frac{\lambda + \xi}{2} \tag{D.8}$$

This is equivalent to $\lambda > \frac{1+\sqrt{1+4\xi(1-\xi)}}{2}$ but $\xi(1-\xi) \leq 1/4$, so (D.8) is equivalent to:

$$\frac{1+\sqrt{1+4\xi(1-\xi)}}{2} \leq \frac{1+\sqrt{2}}{2} \approx 1.207$$

so we conclude that for $\lambda \geq 1.207$ asymmetric beats mixed for all ξ; for $1 < \lambda < 1.207$ it depends on ξ; for $\lambda = 1$ mixed beats asymmetric for all $0 < \xi < 1$.

Lastly we prove Theorem 8. As we said in Section 5.4, for $P < \lambda + \xi$ there is a mixed leaders equilibrium where the common leader plays CC for sure and the group leaders mix between $F^k C^{-k}$ and FF with probability $\tilde{p}$ on the former. We shall refer to it as "the common leader equilibrium" in the sequel. As P crosses a threshold a little above $\lambda + \xi$ the mixed leaders equilibrium is the one described in Theorem 22, where the common leader mixes between CC, FC and CF and the group leaders mix between $F^k C^{-k}$ and FF with a probability $p < \tilde{p}$ on $F^k C^{-k}$. The average group payoff in this equilibrium is higher than in the common leader equilibrium.[18] Unfortunately, the mixing probability p is the root of a cumbersome equation, and it is most easily computed numerically for each set of parameters value. This makes comparisons over varying parameters not convenient. So in the following estimates we use the common leader equilibrium, which is easier although to our disadvantage.

As shown above, see equation (D.5), the highest payoff in the correlated equilibrium is

$$\overline{\pi}^{corr}(\lambda, \xi) \equiv \frac{\xi + (\lambda + \xi)(\lambda - 1)}{\xi + 2(\lambda - 1)}$$

which of course depends on (λ, ξ); notice that it goes to 1 if $\lambda + \xi \to 2$ or $\lambda \to 1$.

On the other hand, the average group payoff in the common leader equilibrium is

$$\tilde{\pi}(\lambda, \xi, P) \equiv (1 - \tilde{p})(1 + \tilde{p}(\lambda + \xi - 1)) = \frac{P + \xi}{(P + \xi + \lambda - 1)^2} \cdot (P + \lambda(\lambda + \xi - 1)).$$

Therefore fixing $\alpha \leq 1$, for each (λ, ξ) there is a threshold that P must reach so that $\tilde{\pi}(\lambda, \xi, P) = \alpha$ - in particular for each (λ, ξ) in the set $\overline{\pi}^{corr}(\lambda, \xi) = \alpha$. To put ourselves in the most unfavorable position, for each α we pick the *highest* P-threshold in the set $\overline{\pi}^{corr}(\lambda, \xi) = \alpha$. Denote this by $P(\alpha)$. By construction, for $P > P(\alpha)$ the average group payoff in the leaders equilibrium is higher than in any correlated equilibrium with average payoff α. We next derive $P(\alpha)$. The level set $\overline{\pi}^{corr}(\lambda, \xi) = \alpha$ describes a curve

$$\xi(\lambda, \alpha) = \frac{(2\alpha - \lambda)(\lambda - 1)}{\lambda - \alpha}.$$

We insert the function $\xi = \xi(\lambda, \alpha)$ describing the α level set of the correlated utility, so that

[18] The reason is that at $\tilde{p}$ the common leader prefers FC and CF to CC; the group leaders raise the probability of FF, to the extent that the common leader's payoff from CC goes up and reaches that from FC and CF (which go down); in the end the group leader's payoff is higher, and so the average group payoff.

the equation $\tilde{\pi}(\lambda, \xi(\lambda, \alpha), P) = \alpha$ implicitly defines a $P(\lambda, \alpha)$; then for each given α we define $P(\alpha)$ as the *highest* such P over λ in the level set $\overline{\pi}^{corr}(\lambda, \xi(\lambda, \alpha)) = \alpha$. It is seen that in fact $P(\alpha)$ corresponds to the point $\lambda = 2\alpha, \xi = 0$ in the correlated payoff α-level set (recall that in Chicken $\lambda \geq 1$ so here $\alpha \geq 1/2$). Then the equality $\tilde{\pi}(\lambda, \xi, P) = \alpha$ becomes

$$\frac{P}{(P + 2\alpha - 1)^2} \cdot (P + 2\alpha(2\alpha - 1)) = \alpha$$

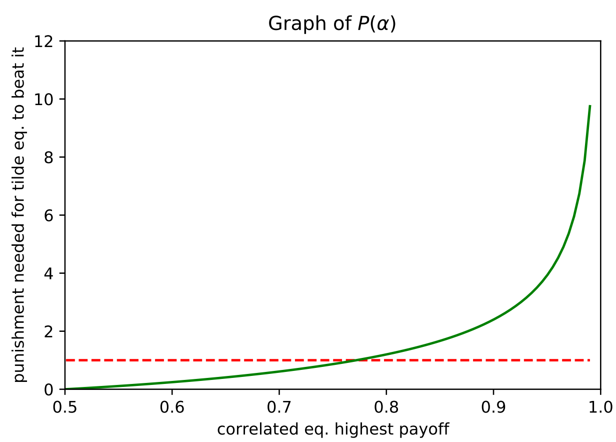
which is seen to be equivalent to

$$P = (2\alpha - 1)\sqrt{\frac{\alpha}{1 - \alpha}} \equiv P(\alpha).$$

The graph of $P(\alpha)$ is in Figure D.2. $P(\alpha) \leq 1$ for $\alpha \lessapprox 0.77$; For $\alpha = 0.9$ it is $P(\alpha) = 2.4$; and for $\alpha = 0.99$ we have $P(\alpha) = 9.75$.

In conclusion, for parameters in the interior of the chicken region what our computations show is that typically, for values of P of the same order of magnitude as the players' payoffs the leaders equilibrium yields higher payoff than any correlated equilibrium of the underlying game.

Figure D.2: **Graph of $P(\alpha)$.** This is the P value above which the mixed leaders equilibrium is higher than *any* correlated equilibrium which gives average group payoff equal to α. Higher values of α are harder to beat. $P(\alpha) \leq 1$ for $\alpha \leq 0.77$; For $\alpha = 0.95$ this is $P(\alpha) = 3.92$; for $\alpha = 0.99$ it is $P(\alpha) = 9.75$. The dashed horizontal line at height 1 is displayed for convenience.



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