Micro II Final Exam: Solutions

Question 1

By definition $x \succ y \iff (x \succsim y) \wedge \neg(y \succsim x)$ so $\Rightarrow$ is evident. It is left to show that $\neg(y \succsim x) \Rightarrow (x \succsim y)$ which by definition $((A \Rightarrow B) \iff (\neg A \vee B))$ is equivalent to $(y \succsim x) \vee (x \succsim y)$; but this is true by definition (comleteness) in weak order.

Question 2

Bob being more risk averse than Ann. By definition we have that for all prospects x and for all outcomes α, if $\alpha \sim_A x \Rightarrow \alpha \succsim_B x$. This is equivalent to $CE_B(x) < CE_A(x)$, so we just take $CE_B(x) < p < CE_A(x)$ and apply definitions.

Question 3

a. In the simultaneous game there are 2 Nash equilibria in pure actions, $(Opera, Opera)$ and $(Game, Game)$ and one in mixed actions $((\alpha_r(Opera) = 2/3, \alpha_r(Game) = 1/3), (\alpha_c(Opera) = 1/3, \alpha_c(Game) = 2/3))$.

b. In the extensive form game with perfect information where the row player (r) moves first, the column player (c) has two information sets: one coinciding with the node reached after r playing *Opera* (h_c^{Opera}) and the other coinciding with the node reached after player r playing *Game* (h_c^{Game}). A strategy for c player is thus a vector of two elements specifying the prescribed action at each information set. Moreover notice that the game has three subgames: one that is the game itself and two that are the sub games originating respectively in the two information sets of player c. Optimality in those two sub games requires that c player chooses *Opera* in (h_c^{Opera}) and *Game* in (h_c^{Game}). The unique SPE of the game is thus $(Opera, (Opera \text{ in } (h_c^{Opera}), Game \text{ in } (h_c^{Game})))$.

c. We begin noticing that the SPE found before is also Nash by construction. Moreover we can find two additional Nash equilibria in pure strategies which are not SPE: $(Opera, (Opera \text{ in } (h_c^{Opera}), Opera \text{ in } (h_c^{Game})))$ and $(Game, (Game \text{ in } (h_c^{Opera}), Game \text{ in } (h_c^{Game})))$. There are also many Nash Equilibria in mixed strategies: they all have in common the fact that player c assigns 0 probability to the strictly dominated pure strategy $(Game, Opera)$. The following sets are all NE in mixed strategies:

   - all the mixed strategy profiles where r plays *Opera* with probability 1 and c plays any mixture between pure strategies $(Opera, Game)$ and $(Opera, Opera)$.
   - all the mixed strategy profiles where r plays *Game* with probability 1 and c plays a mixture between $(Opera, Game)$ and $(Game, Game)$ that assigns a probability smaller or equal to 1/2 to $(Opera, Game)$.

Question 4

a. The mixed Nash equilibria of the simultaneous move game in strategic form are given by the following set of mixed actions' profiles

$$MN := \{(\alpha_{LR}, \alpha_{SR}) \in \Delta(\{+1, -1\}) \times \Delta(\{Out, In\}) : \ \alpha_{LR}(+1) \in [0, 1/2] \quad \text{and} \quad \alpha_{SR}(Out) = 1\}$$

Note that the set MN contains the unique Nash equilibrium $(-1, Out)$ in pure actions which gives the LR player a VNM utility equal to 0. The *minmax* payoff for the LR player is also 0.

b. The pure Stackelberg equilibrium of the extensive game where the LR player moves first is the action profile $(+1, In)$ that gives a VNM utility of 1 to LR.

c. As seen in class, we characterize the set of PPE in terms of the lowest and the highest VNM utility for the LR player, $\underline{v}^1$ and $\bar{v}^1$ respecttively. Given that the static Nash payoff (0) is equal to the *minmax* payoff (see point a.), $\underline{v}^1 = 0$.

In order to determine $\bar{v}^1$ we have to find the worst in the support (WIS):

$\alpha_{LR}(+1)$	BR_2	WIS
0	Out	0
$\in (0, 1/2)$	Out	0
1/2	any mixture of Out and In	≤ 1
$\in (1/2, 1)$	In	1
1	In	1

We conclude that $\bar{v}^1$=1 (the highest WIS) that is equal to the pure Stackelberg payoff.

The minimum value of δ that sustains the pure Stackelberg equilibrium $(+1, In)$ is such that the LR player does not want to deviate from it. In the incentive constraint we have to consider the most tempting deviation for LR, which is -1, and set the continuation payoff equal to the worst dynamic equilibrium payoff (Static Nash payoff (SN)), which is 0.

$$\overbrace{\bar{v}^1}^{=1} = (1-\delta)\overbrace{u_1(-1, In)}^{=2} + \delta w_{LR}(-1)$$

$$\implies w_{LR}(-1) = \frac{1-(1-\delta)2}{\delta} = SN \ (= 0)$$

$$\implies \delta = 1/2$$

For any discount factor greater or equal than 1/2 the LR player has the incentives to sustain the best dynamic equilibrium and the set of PPE payoffs is the closed interval $[0, 1]$.

d. In this case there we have a situation of imperfect public monitoring: the action of LR is not perfectly observed by SR players that observes noisy signals. Let us call the signals y_+ and y_-. The good signal y_+ is observed with probability $(1-\epsilon)$ if the LR player plays $+1$ while it is observer with probability ϵ if the LR player plays -1. Following the notation introduced in class we write the probability of the observed outcome (signal) as a function of the action profile as

$$\rho(y_+|a) = \begin{cases} 1-\epsilon & \text{if } a_1 = +1 \\ \epsilon & \text{if } a_1 = -1 \end{cases} \qquad \rho(y_-|a) = \begin{cases} \epsilon & \text{if } a_1 = +1 \\ 1-\epsilon & \text{if } a_1 = -1 \end{cases}$$

We want to find the best equilibrium payoff $\bar{v}^1_\epsilon$ for LR in this environment. First we notice that it has to be that $\bar{v}^1_\epsilon \in [\underline{v}^1, \bar{v}^1] = [0, 1]$. Second, we know by public perfect randomization that the set of equilibrium payoffs will be a compact set (closed line interval $[\underline{v}^1_\epsilon, \bar{v}^1_\epsilon]$). In particular the lower bound will still be 0: this payoff is clearly enforceable through the repetition of the static Nash.

Let us now define the continuation payoff as function of the signals

$$w : \{y_+, y_-\} \to [0, \bar{v}^1_\epsilon]$$

Looking for the best equilibrium payoff we have to enforce $(+1, In)$.

$$\bar{v}^1_\epsilon = (1-\delta)\overbrace{u_1(+1, In)}^{=1} + \delta[(1-\epsilon)w(y_+) + \epsilon w(y_-)] \tag{1}$$

In order to do this the continuations have to satisfy the following dynamic incentive constraint

$$(1-\delta)1 + \delta[(1-\epsilon)w(y_+) + \epsilon w(y_-)] \geq (1-\delta)\overbrace{u_1(-1, In)}^{=2} + \delta[\epsilon w(y_+) + (1-\epsilon)w(y_-)]$$

Rearranging we get

$$w(y_+) \geq w(y_-) + \frac{1-\delta}{\delta(1-2\epsilon)} \tag{2}$$

Notice that the continuation payoff difference shrinks as δ increases (as the temptation of current deviation diminishes) and as ϵ decreases (as the observed outcome becomes more responsive to actual play). Given imperfect monitoring, we have to choose the highest possible reward for LR when there is a big probability that she did good, i.e. we have to impose

$$w(y_+) = \bar{v}^1_\epsilon \tag{3}$$

But we also have to choose the highest possible continuation when the bad signal is observed but still there is some probability that LR played $+1$, i.e. we have to choose the smallest punishment. In order to do this we impose (2) holding as an equality.

Combining (1), (2) holding as an equality and (3) we get

$$\bar{v}^1_\epsilon = 1 - \frac{\epsilon}{1-2\epsilon}$$

This is the highest payoff that we can get for δ high enough.

Notice that $\bar{v}^1_\epsilon \leq \bar{v}^1$: this is the inefficiency due to moral hazard.

Moreover $\bar{v}^1_\epsilon \geq \underline{v}^1 \iff \epsilon \leq 1/3$. For $\epsilon \geq 1/3$ the only enforceable equilibrium is the static Nash.