

Final Exam Answers: Economics 101

December 8, 1997 © David K. Levine

1. Normal Form Games (note that a complete answer must include a drawing of the socially feasible sets)

a)

	L	R
U	2*,5*	0,0
D	0,0	5*,2*

Two pure strategy equilibria as marked. Mixed for player 2 $2p = 5(1-p)$ so $p=5/7$; for player 1 $5q = 2(1-q)$ so $q=2/7$. Pure strategy equilibria are Pareto Efficient. The mixed equilibrium is not. No weakly dominated strategies. Pure strategy maxmin is 0; pure strategy minmax is 2; mixed strategy maxmin for player 1 must satisfy $2q = 5(1-q)$ so $q = 5/7$ and the maxmin is $2(5/7) = 10/7$.

b)

	L	R
U	-1*,1	-3,3*
D	-3,3*	-1*,1

No pure strategy equilibrium. Unique pareto efficient mixed equilibrium where both players mix 50-50. No weakly dominated strategies. Note that the socially feasible set is one-dimensional. Pure strategy maxmin for player 1 is -3 , for player 2 is 1 ; pure strategy minmax for player 1 is -1 , for player 2 is 3 . mixed strategy maxmin is achieved by playing 50-50; for player 1 -2 for player 2 $+2$.

c)

	L	R
U	7,7	0,8*
D	8*,0	1*,1*

Unique Nash equilibrium (U,L are strictly dominated). No mixed equilibria. Nash equilibrium is not pareto efficient. Pure and mixed maxmin and maxmin is 1 for both players.

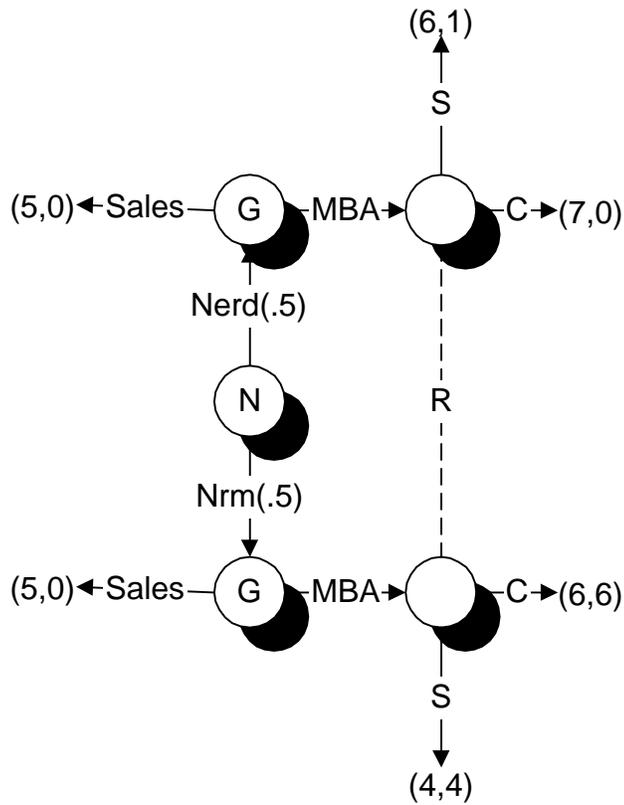
2. Long Run versus Short Run

	L	R
U	3,1*	0,0
D	8*,0	1*,2*

The unique Nash equilibrium is DR; the Stackelberg equilibrium is UL. Strategies for which lead to playing UL are UL if always UL in the past and DR if ever a deviation. Alternatively, players may base their strategies on past play of the LR player only: LR: U if U in the past and D if ever a deviation by LR and SR: L if U in the past and R if ever a deviation of the LR player.

These are optimal for the short-run player because it is in his best-response correspondence. For the long run player it must be that $3 \geq (1 - \delta)8 + \delta 1$ or $\delta \geq 5/7$.

3. Screening



Nerd/Nrm	S	C
SS	5,0*	5,0*
SM	4.5,2	5.5,3*
MS	5.5*,0.5*	6,0
MM	5,2.5	6.5*,3*

Two pure equilibria: MS and S; MM and C. Note that for the graduate SS and MS are strictly dominated by MS. So we look for the randomization by R that makes graduate indifferent between MS and MM: $5.5p + 6(1 - p) = 5p + 6.5(1 - p)$ or $p = 0.25$. Then we look for the randomization between MS and MM that makes R indifferent between S and C. $0.5q + 2.5(1 - q) = 0q + 3(1 - q)$ or $q = 0.5$. The corresponding behavior strategy (only Graduate has a difference between behavior and mixed strategy) is get and MBA if nerd

and choose MBA with probability 0.5 if normal. Beliefs of recruiter are then 2/3 nerd, 1/3 normal.

4. Decision Analysis

without the test payoff from banning all athletes $.1 \times 0 + .9 \times (-100) = -90$; payoff from allowing all athletes to participate $.1 \times (-50) + .9 \times 10 = 4$, so allow all to participate and get payoff of 4.

Test positive probability of user by Bayes law

$$pr(user|+) = \frac{.95 \times .1}{.95 \times .1 + .1 \times .9} = .51 \quad pr(+) = .95 \times .1 + .1 \times .9 = .185$$

$$pr(user|-) = \frac{.05 \times .1}{.05 \times .1 + .9 \times .9} = .006 \quad pr(-) = .05 \times .1 + .9 \times .9 = .815$$

payoff to + and ban $.51 \times 0 + .49 \times (-100) = -49$; payoff to + and participate $.51 \times (-50) + .49 \times 10 = -20.6$ so ban and get payoff of -20.6

payoff to – and ban is obviously negative

payoff to – and participate is $.006 \times (-50) + .994 \times 10 = 9.64$

overall utility if test is used optimally $.185 \times (-20.6) + .815 \times 9.64 = 4.05$

gain to using test $4.05 - 4 = .05$, so pay up to $.05$ per athlete.

Erratum: the answer key is wrong. The first mistake is just a typo, it says that the payoff from ban is -49 and from participate is -20.6, so "ban" and get payoff of -20.6 (it should say so " don't ban" and get a payoff of -20.6). The main problem however is that the problem was done rounding the payoffs and probabilities yielding a solution of being willing to pay up to \$.05 for the test when if done with "all the decimals" you'd get that you would not pay a cent. Just from intuition the answer should be zero, since having the test is not changing our decisions (we are not banning any way). Enrique Flores

5. Cournot with Uncertain Cost

$$\pi_i(x_i, c_i) = (1/3)[17 - c_i - (x_i + x^1)]x_i \\ + (2/3)[17 - c_i - (x_i + x^3)]x_i$$

maximize

$$\frac{d\pi_i(x_i, c_i)}{dx_i} = [17 - c_i - (2x_i + (1/3)x^1 + (2/3)x^3)] = 0$$

$$\text{so } 2x_i = (17 - c_i - (1/3)x^1 - (2/3)x^3)$$

$$2x_i = (17 - c_i - (1/3)x^1 - (2/3)x^3)$$

$$6x_i = (51 - 3c_i - x^1 - 2x^3)$$

solve each equation individually

$$7x^1 = 48 - 2x^3$$

$$8x^3 = 42 - x^1 \text{ or } x^3 = 21/4 - x^1/8$$

plug the second into the first

$$7x^1 = 48 - 2(21/4 - x^1/8) = 75/2 + x^1/8 \text{ or } x^1 = 60/11$$

substitute back to get $x^3 = 201/44$

Erratum: the answer key is wrong. $7x^1 = 48 - 2(21/4 - x^1/8) = 75/2 + x^1/4$ so

$x^1 = 50/9$. *Substituting back in we get* $x^3 = 143/36$.

Mark Fann