Answers to Problem Set 3: Dynamic Game Theory

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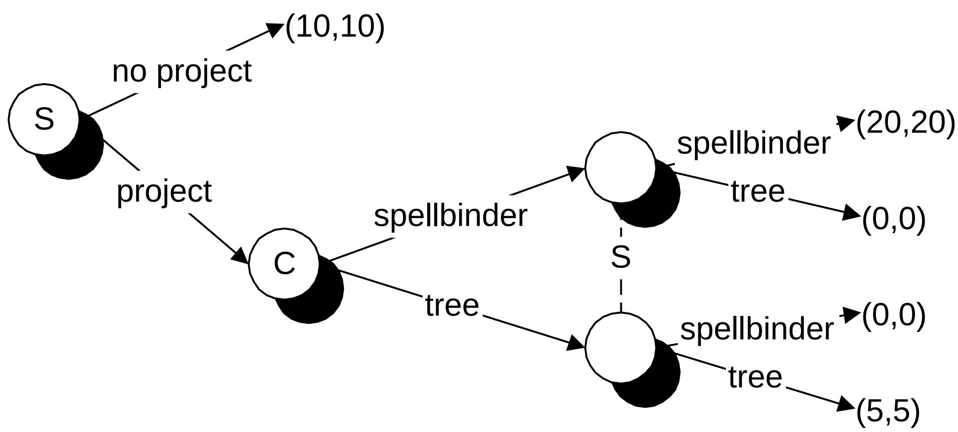
1. Forward Induction

extensive form



normal form with reaction function and Nash equilibria marked

	spellbinder	tree
no: spellbinder	10*,10	10*,10*
no: tree	10*,10	10*,10*
yes: spellbinder	20*,20*	0,0
yes: tree	0,0	5*,5

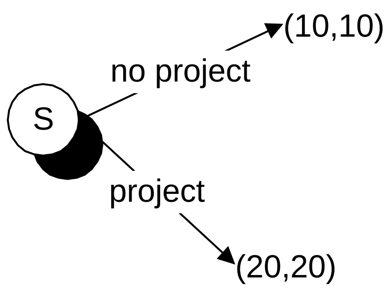
to find subgame perfect equilibria, must first find the subgames: there are two; one is the entire game, the other is the game that begins with C's move

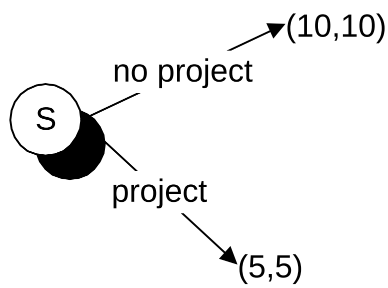
[NOTE: there is a second correct extensive form in which the subgame begins with S's move]

The normal form of this subgame is

	spellbinder	tree
spellbinder	20*,20*	0,0
tree	0,0	5*,5*

As shown there are two Nash equilibria. We must therefore draw two different game trees in each case replacing the subgame with the Nash payoffs





In the first case, the equilibrium is 20,20; in the second case it is 10,10. These are the same as the Nash equilibria.

For iterated weak dominance, we return to the normal form (with the first two strategies combined)

	spellbinder	tree
no	10,10	10,10
yes: spellbinder	20,20	0,0
yes: tree	0,0	5,5

no strategy is weakly dominated for player 2; however, the strategy of yes: tree is weakly dominated for player 1 by no. This gives the reduced game

	spellbinder	tree

no	10,10	10,10
yes: spellbinder	20,20	0,0

Now Spellbinder weakly dominates tree for player 2 giving

	spellbinder
no	10,10
yes: spellbinder	20,20

Now yes: spellbinder weakly dominates no, so that the only thing left after iterated weak dominance is that Stephen begins the project, and they agree on Spellbinder.

2. The Folk Theorem

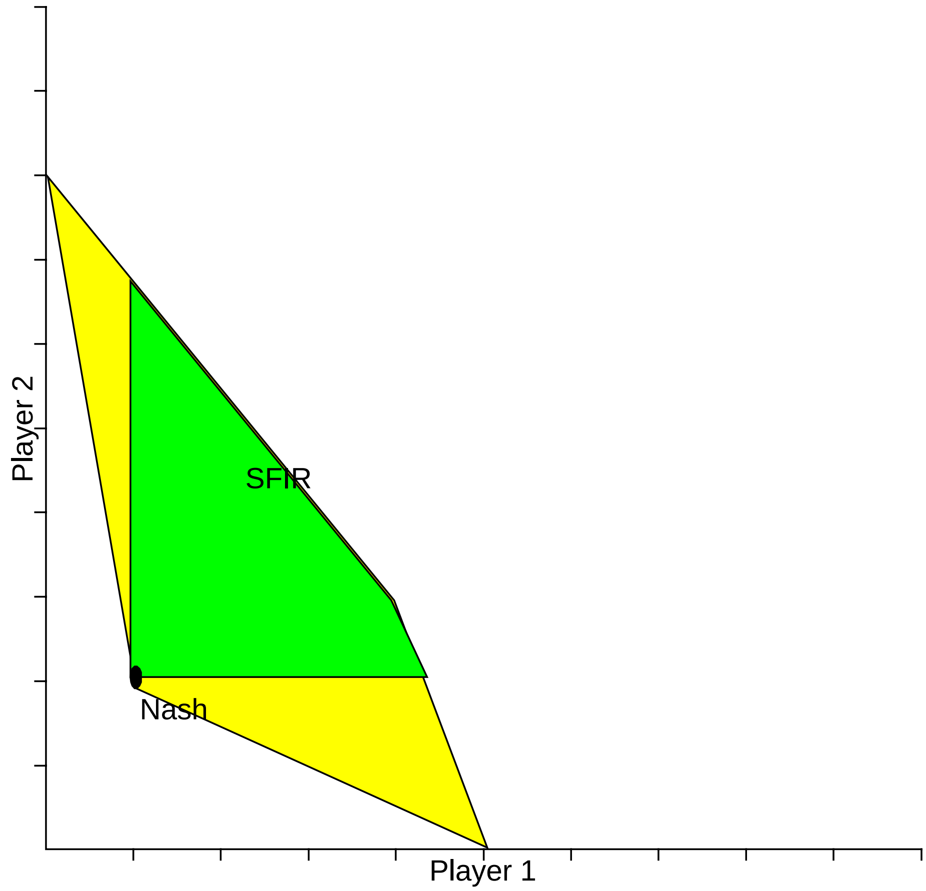
a)

	L	R
U	4,3	0,7*
D	5*,0	1*,2*

Dominant strategies so no mixed equilibrium

Minmax for 1 is 1 by playing D

Minmax for 2 is 2 by playing R

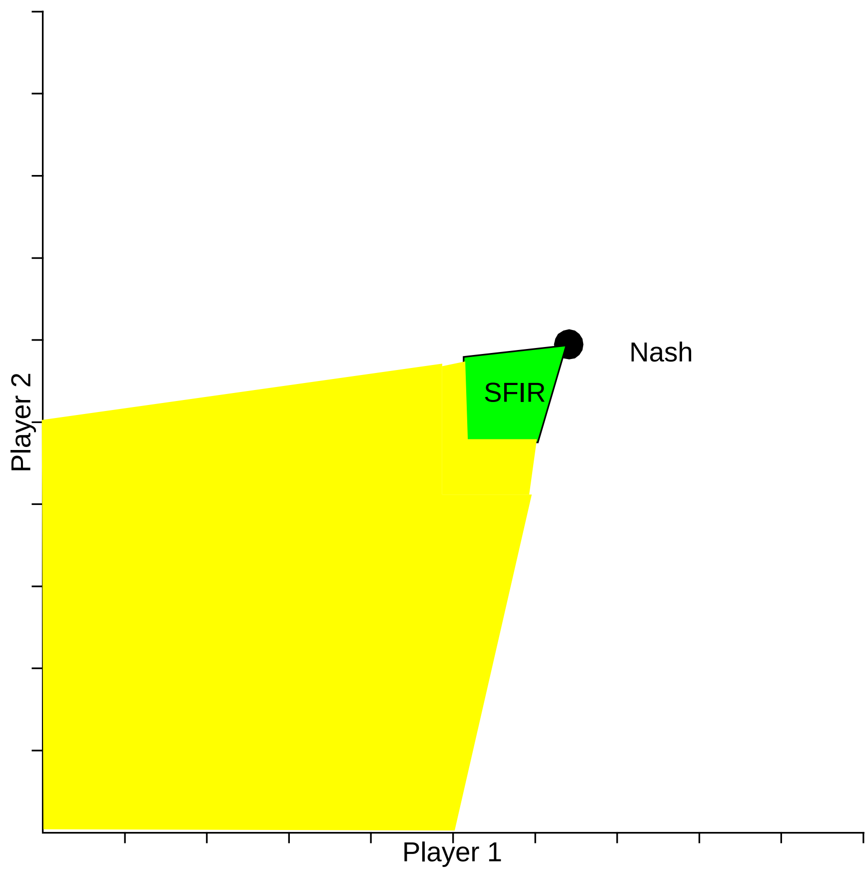


b)

	L	R
U	6*,6*	5*,0
D	0,5*	0,0

Dominant strategy so no mixed equilibrium

Minmax for both players is 5



Nash
SFIR
Player 2
Player 1

3. Equilibrium in a Repeated Game

	U	D
U	1,1	-1,100
D	100,-1	0,0

If you play U against grim always you get an average present value of 1

1. If you play D against grim you get $(1-\delta)100$ in the first period and 0 (or –1) in every subsequent period. So it must be that $1 \geq (1-\delta)100$ or $\delta \geq .99$.