

## Final Exam: Economics 101

You have three hours. Do all 5 questions; each has equal weight. Please be sure to number each problem by number and part, especially if you choose to do them out of order. Good luck.

December 17, 1997 © David K. Levine

### 1. Normal Form Games

In each of the following games

- i) Find all of the pure strategy Nash equilibria
- ii) Determine whether or not there is a mixed strategy Nash equilibrium, and if so, what it is
- iii) Which of these equilibria are Pareto efficient?
- iv) Do any of the pure strategy equilibria involve playing weakly or strictly dominated strategies?
- v) Sketch the socially feasible set.
- vi) Find a pure strategy that maximizes the payoff each player is guaranteed no matter how the other player plays (this is called the maxmin payoff). Also find a pure strategy that minimizes the payoff the other player can get (this is called the minmax payoff). Sketch the corresponding socially feasible individually rational set.

a)

	L	R
U	2,5	-1,1
D	1,-1	5,2

b)

	L	R
U	5,5	-1,8
D	8,-1	1,1

c)

	L	R
U	-1,3	-3,5
D	-3,5	-1,3

## **2. Repeated Games**

	L	R
U	3,3	0,5
D	5,0	1,1

Suppose that this stage game is repeated between two infinitely lived players with discount factor equal to  $\delta$ . Propose a strategy and a discount factor  $\delta$  such that the equilibrium outcome of the game is for both players to play UL.

## **3. Long Run versus Short Run**

	L	R
U	2,1	0,0
D	11,0	1,3

Suppose that this stage game is repeated between infinitely-lived player 1 (row player) with discount factor equal to  $\delta$  and a sequence of short-lived player 2's (column players).

What pure strategy Nash equilibria are in the stage game? What is the Stackelberg equilibrium of the stage game in which player 1 moves first? Propose a strategy and a discount factor  $\delta$  such that in equilibrium players end up playing UL.

#### **4. Screening**

A recent college school graduate must decide whether to go get an MBA or to continue working as a car salesperson for a utility of 5. With probability .9 the graduate is a nerd, and with probability .1 he is a surfer dude. He knows whether or not he is a nerd, but business school recruiters do not. It costs 1 unit of utility to go to business school. The business school recruiter must decide whether to offer a straight contract, or an incentive contract in which extra hard work is repaid by extra time off. The nerd gets a utility of 8 from the straight contract (don't forget the cost of getting the MBA though) and a utility of 5 from the incentive contract, since it just means he has to work harder, and time off has no value to a nerd. On the other hand, the surfer dude gets a utility of 5 from the straight contract, but gets 8 from the incentive contract, since he can work hard, and spend the extra time surfing. The recruiter gets a bonus of 1 from his employer if he can hold labor costs down to 5, otherwise he gets nothing.

- a) Draw the extensive form of the game.
- b) Find the normal form.
- c) What are the pure strategy Nash equilibria?
- d) What is the mixed strategy Nash equilibrium?
- e) In this mixed strategy equilibrium, what are the beliefs of the recruiter about the type of MBA graduate he gets? (You will need to use Bayes law to answer this.)

#### **5. Price Discrimination**

You may sell either 1 or 2 units of a good to a consumer. (You may not sell 0 units, and there is no production cost.) You know that this consumer is one of two types type H

(high demand) and type L (low demand), and that both types are equally likely (probability 1/2). The high demand type has utility function  $(5-p)x$ , and the low demand type has utility function  $(3-p)x$ , where  $p$  is the price paid per unit and  $x$  is the number of units purchased. You do not know the consumer's utility function, but the consumer does. So you choose to have the consumer play a "demand revelation game." In this game the strategy of the consumer is to announce either that he is type H or type L. (This is a one-person game.) You choose how the price and quantity depend on the announcement; that is, you choose four numbers  $p^H, x^H, p^L, x^L$ , where  $x^H, x^L$  must be 1 or 2 units (the only amounts you are allowed to sell).

- a) For what values of  $p^H, x^H, p^L, x^L$  is it an optimal strategy for the consumer to tell the truth?
- b) For what values of  $p^H, x^H, p^L, x^L$  is it an optimal strategy for a truth-telling consumer to enter the game? (He gets utility zero if he decides not to buy from you.)
- c) If you restrict yourself to choosing  $p^H, x^H, p^L, x^L$  so that the consumer chooses to buy from you and to tell the truth, what values of  $p^H, x^H, p^L, x^L$  maximize your expected revenue? Be sure to consider all possible cases, bearing in mind that you may sell 1 unit or 2 but not 0 units.