

Copyright (C) 2001 David K. Levine

This document is an open textbook; you can redistribute it and/or modify it under the terms of version 1 of the open text license amendment to version 2 of the GNU General Public License. The open text license amendment is published by Michele Boldrin et al at <http://levine.sscnet.ucla.edu/general/gpl.htm>; the GPL is published by the Free Software Foundation at <http://www.gnu.org/copyleft/gpl.html>.

Iterated Dominance in the Cournot Model

weak dominance never a lower payoff no matter what the opponent does, and sometimes a higher payoff

strict dominance a higher payoff no matter what the opponent does

admissibility: never use a weakly dominated strategy

If weakly dominated strategies are not used, should players anticipate that opponents will not use them?

Iterated weak dominance: eliminate weakly dominated strategies to get a smaller game, then repeat this procedure

Example of Iterated Weak Dominance

	L	M	R
U	-1,-1	2,0	1,1
M	-1,-1	1,-1	0,0
D	1,1	1,1	1,2

Eliminate M, weakly dominated by U

	L	M	R
U	-1,-1	2,0	1,1
D	1,1	1,1	1,2

Eliminate L, weakly dominated by R

	M	R
U	2,0	1,1
D	1,1	1,2

Eliminate D, weakly dominated by U

	M	R
U	2,0	1,1

Eliminate M, strictly dominated by R

	R
U	1,1

An alternative procedure

	L	M	R
U	-1,-1	2,0	1,1
D	1,1	1,1	1,2

Eliminate L **AND** M, weakly dominated by R

	R
U	1,1
D	1,2

can proceed no further

Problems with Iterated Weak Dominance

- procedure is ambiguous
- it may yield more than one answer
- it is not “robust”

Robustness

To avoid playing a weakly dominated strategy, a player must know his own payoffs exactly.

To know that his opponent is not playing a weakly dominated strategy, a player must know his opponent's payoffs exactly. This is a very strong assumption.

To know that his opponent is not playing a strictly dominated strategy, a player must only know his opponent's payoffs approximately.

A plausible (and robust) concept: iterated strict dominance, or the stronger notion of $S^\infty W$

Iterated Strong Dominance and Duopoly

$$p = a - bx$$

$$a = 17, c = 1, b = 1$$

so that the competitive solution is 16 units of output and the monopoly solution is 8 units of output

profits

$$\begin{aligned}\pi_i &= [17 - (x_i + x_{-i})]x_i - x_i \\ &= [16 - (x_i + x_{-i})]x_i\end{aligned}$$

possible output levels 0, 4, 5, 8, 12, 16

	0	4	5	8	12	16
0	0,0	0,48	0,55	0,64*	0*,48	0*,0
4	48,0	32,32	28,35*	16*,32	0*.0	-16,-64
5	55,0	35*,28	30*,30*	15,24	-5,-12	-25,-80
8	64*,0	32,16*	24,15	0,0	-32,-48	-64,-128
12	48,0*	0,0*	-12,-5	-48,-32	-96,-96	-144,-192
16	0,0*	-64,-16	-80,-25	-128,-64	-192,-144	-256,-256

The *'s mark the *best response* or *reaction* function

Iterated Strict Dominance

	0	4	5	8	12	16
0	0,0	0,48	0,55	0,64*	0*,48	0*,0
4	48,0	32,32	28,35*	16*,32	0*.0	-16,-64
5	55,0	35*,28	30*,30*	15,24	-5,-12	-25,-80
8	64*,0	32,16*	24,15	0,0	-32,-48	-64,-128
12	48,0*	0,0*	-12,-5	-48,-32	-96,-96	-144,-192
16	0,0*	-64,-16	-80,-25	-128,-64	-192,-144	-256,-256

	0	4	5	8
0	0,0	0,48	0,55	0,64*
4	48,0	32,32	28,35*	16*,32
5	55,0	35*,28	30*,30*	15,24
8	64*,0	32,16*	24,15	0,0

	4	5	8
4	32,32	28,35*	16*,32
5	35*,28	30*,30*	15,24
8	32,16*	24,15	0,0

	4	5
4	32,32	28,35*
5	35*,28	30*,30*

Continuous Case

Suppose that BMG expects that CBA will produce x_{-i} units of output.
What should BMG do?

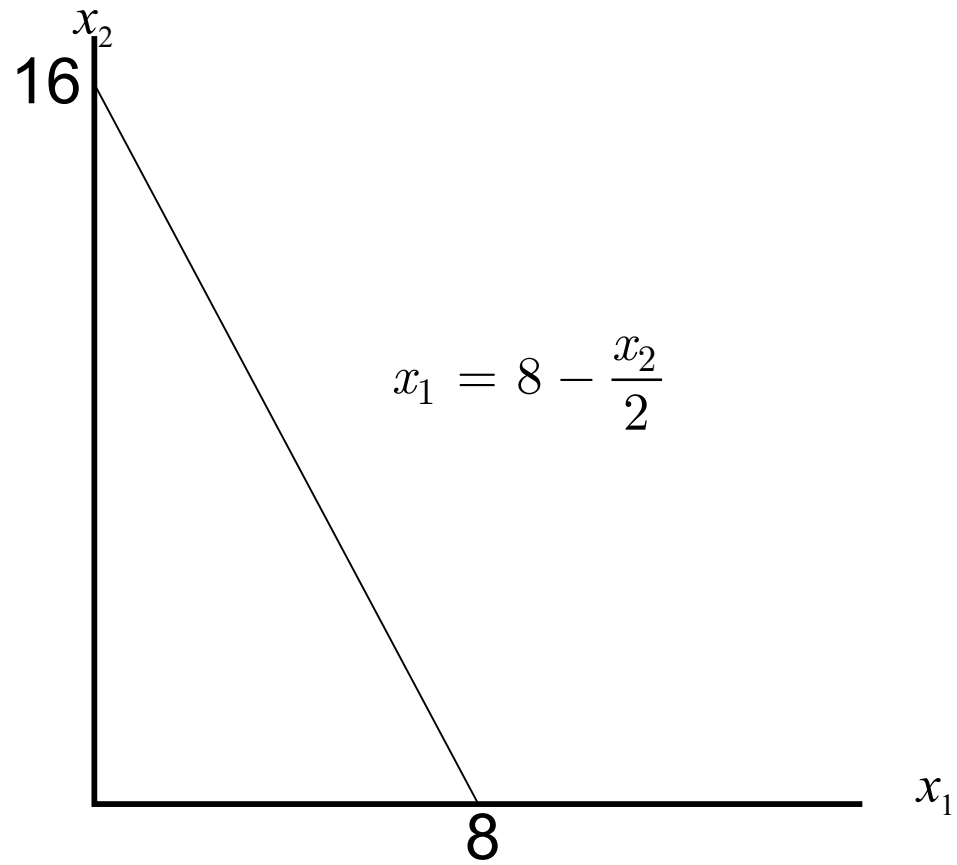
$$\pi_i = [17 - (x_i + x_{-i})]x_i - x_i$$

$$\frac{d\pi_i}{dx_i} = 16 - 2x_i - x_{-i} = 0$$

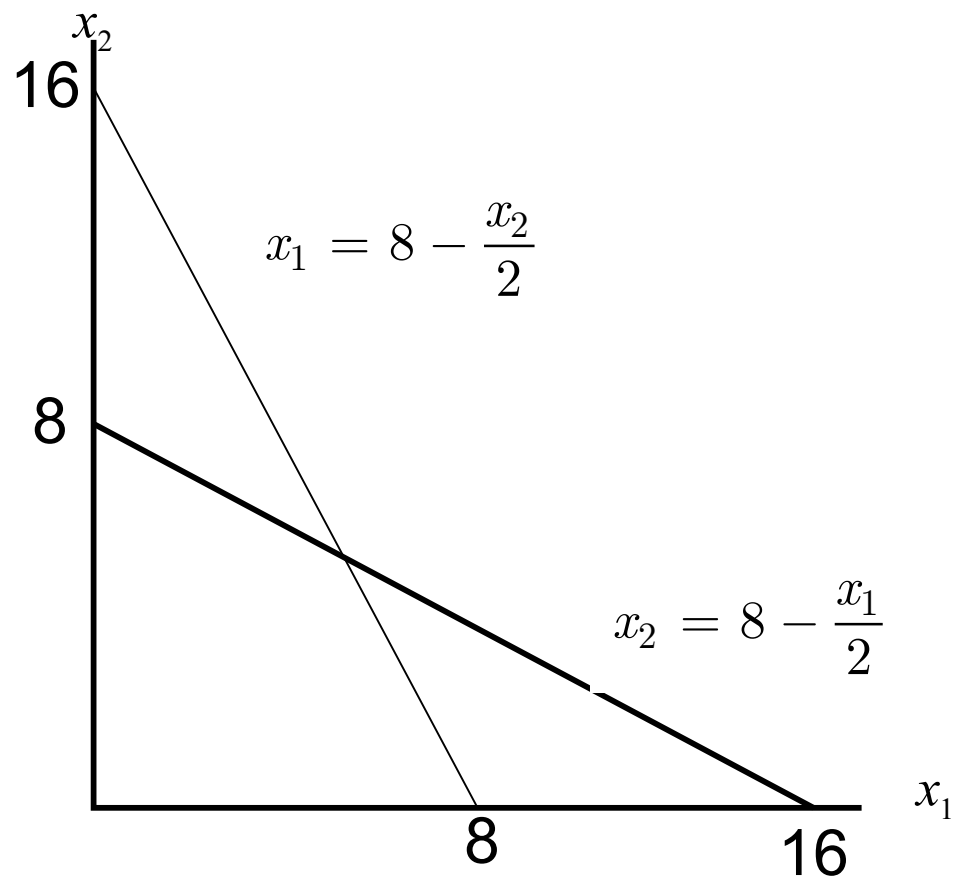
solving we find

$$x_i = 8 - \frac{x_{-i}}{2}$$

This is called the *best response* or *reaction* function of BMG to CBA.



The Cournot Equilibrium



$$x = \frac{16 - x}{2}$$

$$x = \frac{16}{3}$$

- less than monopoly but more than half monopoly
- industry output is twice this amount
- this is $\frac{2}{3}$ the competitive output, as against $\frac{1}{2}$ for monopoly