PRODUCTION THEORY

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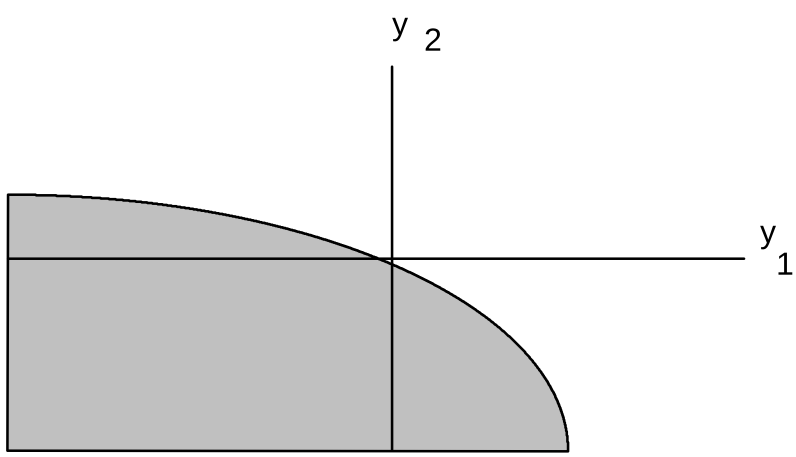
The Production Set

y_i amount of the i th input or output (+for output, - for input)

Y production set; we write

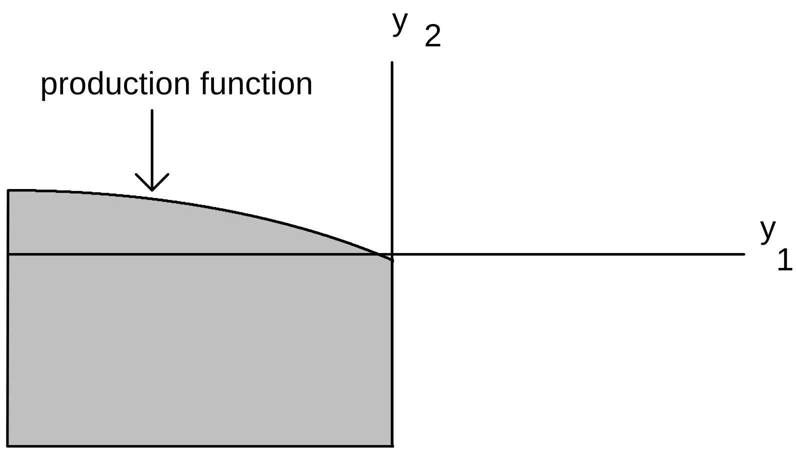
$y \in Y$ if $y = (y_1, y_2, \ldots, y_m)$ is a feasible production vector



Note that ordinarily $0 \in Y$

The Production Function

Special case: y_1 is an output , $y_2, \ldots, y_m$ are inputs



the production function is written $y_1 = f(-y_2, -y_3, \ldots, -y_m)$

or $y_1 = f(x_2, x_3, \ldots, x_m)$ where the x are positive

Example of the Cobb-Douglas production function

$y_1 = A(x_2)^{\alpha}(x_3)^{\beta}$

Cost Minimization Problem

$$\min_{x_2, x_3} \quad p_2 x_2 + p_3 x_3$$

subject to $y_1 \leq f(x_2, x_3)$

The solution are the *conditional demand functions* together with the *cost function*

$x_i(y_1, p_2, p_3)$

$C(y_1, p_2, p_3)$

Properties of Cost and Derived Demand

(1) Demand is homogeneous of degree zero in prices $x_i(y_1, \lambda p_2, \lambda p_3) = x_i(y_1, p_2, p_3)$
Cost is homogeneous of degree one in prices $C(y_1, \lambda p_2, \lambda p_3) = \lambda C(y_1, p_2, p_3)$

(2) $C(y_1, p_2, p_3) = p_2 x_2(y_1, p_2, p_3) + p_3 x_3(y_1, p_2, p_3)$

(3) Shephard's lemma $x_i(y_1, p_2, p_3) = \dfrac{\partial C(y_1, p_2, p_3)}{\partial p_i}$

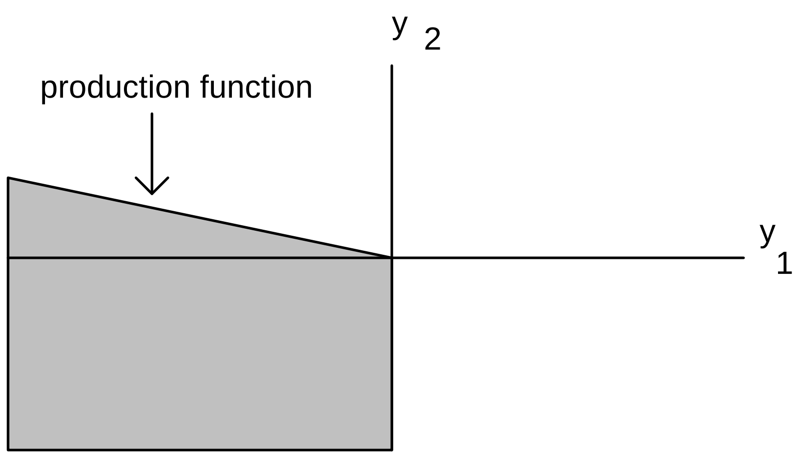
Marginal Cost and Returns to Scale

marginal cost is $MC \equiv \dfrac{\partial C(y_1, p_2, p_3)}{\partial y_1}$

increasing returns to scale if marginal cost decreases with output
constant returns to scale if marginal cost does not change as output changes
decreasing returns to scale if marginal cost increases with output

We ordinarily assume either decreasing or constant returns to scale, and competition ought to mean constant returns to scale, at least at the level of the industry.

Constant returns in the production set



Profit Maximization

$$\max_{y_1, x_2, x_3} \quad p_1 y_1 - (p_2 x_2 + p_3 x_3)$$

subject to $\quad y_1 \le f(x_2, x_3)$

To maximize profits, you must minimize costs, so this is the same as

$$\max_{y_1} \quad p_1 y_1 - C(y_1, p_2, p_3)$$

First order condition is

$$p_1 = \frac{\partial C(y_1, p_2, p_3)}{\partial y_1}$$

or *price equals marginal cost* or the marginal cost curve is the supply curve