Economics 201B – Midterm Exam Solutions, Winter 2002

Question one: Game Theory (20 points)

Find:

1) the normal form (2 points)

	u	d
I	5, 5	0, 0
O	3, 7	3, 7

2) all dominant strategy equilibria (1 point)

None, because no dominant strategy for either player.

3) apply iterated weak dominance (2 points)

'd' is weakly dominated by 'u' for player two. Once it is eliminated, 'O' is strictly (and hence also weakly) dominated by 'I' for player one. Thus the game is solvable by IEWDS, giving $\{(I,u)\}$.

4) all pure and mixed Nash equilibria (3 points)

$\{(I,u),(O,u$ played with probability $\le 3/5)\}$

5) those Nash equilibria which are THP (3 points)

$\{(I,u)\}$ is strict and hence THP. All the others, involving the play of a weakly dominated strategy, are not THP by definition.

6) all subgame perfect equilibria (2 points)

A game of perfect information without ties has a unique SGPE found by backward induction, in this case $\{(I,u)\}$.

7) all heterogeneous SCE (3 points)

For two player games of perfect information, the set of heterogeneous SCE is the same as the set of public randomizations over Nash Equilibria. More explicitly, Agent 1 in the role of player 1 believes player 2 will play 'd', so he plays 'O' and his belief is never contradicted, and player 2 plays whatever, since it doesn't matter what player 2 plays. Agent 2 in the role of player 1 believes player 2 will play 'u', so player 1 plays 'I' and player 2 does indeed play 'u' (since player 2 is rational and faces a pure decision problem), so player 1's beliefs are confirmed at the information set that is reached. Any mixture of agent 1s and agent 2s in the large population playing the role of player 1 will give any mixture over the two pure strategy equilibria.

8) a correlated equilibrium that is not a public randomization over Nash Equilibria, or indicate why there is none. (4 points)

The set of all correlated equilibria payoff profiles (in payoff space) coincides with that obtained by all public randomizations over all Nash equilibria.

Question two: An "Auction" (20 points)

1) What is the optimal take-it or leave-it price? (8 points)

Consider the three cases: If the seller offers $p > 10$, then no type buys and the seller gets zero profit for sure. If the seller offers a price between $4 < p \leq 10$ then the low type doesn't buy but there is a 50% chance the high type does buy, so a price of 10 (no sense offering less) gives an expected profit of 5. If the seller offers $p \leq 4$ then both types will buy and the seller sells for sure, so the highest obtainable profit is this case is 4 when 4 is offered (no sense offering less). Hence the 'optimal' (for the seller that is) price is 10, for a profit of 5.

2) Find a pair of take it or leave it lotteries that yield the seller higher expected revenue than the optimal take-it or leave-it price? (12 points)

In part one we were constrained to offering one price. We want to be able to separate out the two types. To hit two targets you need (at least) two arrows. Two prices alone won't do it, since the high type will always prefer the low price. But we can exploit the fact that the high type is risk averse (strictly concave utility function) and the low type is risk neutral (linear utility function). Thus the low type only cares about the relative expected values of the two lotteries the seller offers, while the high type cares about that <u>and</u> the relative risk (variance) of the two lotteries.

The question asks for <u>a</u> pair of lotteries, and any pair that gives the seller a profit between 5 and 7 (the first best profit) was acceptable (the question did not ask for the optimal mechanism for the seller - obtainable only in the limit anyway). For example, consider the following two lotteries:

$L_1 = (7,0,1,0)$ for an expected value of 7, and
$L_2 = (10,-2,0.5,0.5)$ for an expected value of 4.

So the low type prefers the second lottery and the high type the first ($EU_H(L_1) \approx 0.602$ compared to $EU_H(L_2) \approx 0.557$), giving the seller a profit of 5.5>5. The two types have been separated out.