Decision Theory

- Rational individual choice is the central underpinning of economic theory
- Do people choose rationally? What do we mean by rationally?

Expected Utility

Luce, D. and H. Raiffa [1957]: *Games and Decisions,* John Wiley chapter 2.5

there are r prizes $1,\ldots,r$

a lottery L consists of a finite vector $(p_1,\ldots,p_r)$ where p_i is the "probability" of winning prize i

properties of "probabilities" $p_i \geq 0, \sum_{i=1}^{r} p_i = 1$

Definition: the lottery L_i has $p_i = 1$

Preferences $\geq$ are defined over the set of lotteries

order the lotteries so that $L_i \geq L_{i+1}$, that is higher numbered prizes are worse

Usual preference assumptions:

1) transitivity

2) continuity: for each L_i there exists a lottery $\tilde{L}_i$ such that $p_j = 0$ for $j = 2, \ldots, r-1$ and $L_i \sim \tilde{L}_i$

(in words: we can find probabilities of the best and worst prize that are indifferent to any lottery)

Definition: u_i is such that $\tilde{L}_i = (u_i, 0, \ldots, 0, (1 - u_i))$

Assumptions relating to probability:

a compound lottery is a lottery in which the prizes are lotteries

we can write a compound lottery $(q^1, L^1, q^2, L^2, \ldots, q^k, L^k)$ where

q^i is the probability of lottery L^i (not to be confused with L_i)

1) reduction of compound lotteries

preferences are extended from simple lotteries to lotteries over lotteries by the usual laws of probability

example: $L^1 = (p_1^1, p_2^1, \ldots, p_r^1)$, $L^2 = (p_1^2, p_2^2, \ldots, p_r^2)$

$(q_1, L^1, q_2, L^2) \sim (q^1 p_1^1 + q^2 p_1^2, q^1 p_2^1 + q^2 p_2^2, \ldots, q^1 p_r^1 + q^2 p_r^2)$

2) substitutability (independence of irrelevant alternatives)

for any lottery L the compound lottery that replaces L_i with $\tilde{L}_i$ is indifferent to L

$(p_1, p_2, \ldots, p_r) \sim (p_1, L_1, p_2, L_2, \ldots, p_i, \tilde{L}_i, \ldots, p_r, L_r)$

3) monotonicity

$(p, 0, \ldots, 0, (1-p)) \geq (p', 0, \ldots, 0, (1-p'))$ if and only if $p \geq p'$

Expected utility theory:

Start with a lottery $L = (p_1, \ldots, p_r)$

Using transitivity and continuity L is indifferent to the compound lottery $(p_1 \tilde{L}_1, \ldots, p_r \tilde{L}_r)$

Notice that the lotteries $\tilde{L}_i$ involve only the highest and lowest prizes

Now apply reduction of compound lotteries: this is equivalent to the lottery

$L \sim (u, 0, \ldots, 0, (1-u))$ where $u = \sum_{i=1}^{r} p_i u_i$

This says that we may compare lotteries by comparing their “expected utility” and by monotonicity, higher utility is better

Allais Paradox

Take Q = 1 billion dollars US

Decision problem 1:

Q for sure

(or)

.1 x 5Q, .89 x 1Q, .01 x 0Q

Decision problem 2:

.1 x 5Q, .9 x 0Q

(or)

.11 x 1Q, .89 x 0Q

Decision problem 1:

1 x 1Q for sure **[most common choice]**

(or)

.1 x 5Q, .89 x 1Q, .01 x 0Q

Decision problem 2:

.1 x 5Q, .9 x 0Q **[most common choice]**

(or)

.11 x 1Q, .89 x 0Q

So $u(1) > .1u(5) + .89u(1) + .01u(0)$ or $u(5) < 1.1u(1) - .1u(0)$

And $.1u(5) + .9u(0) > .11u(1) + .89u(0)$ or $u(5) > 1.1u(1) - .1u(0)$

Notice that the original problem had Q equal to 1 million US. This doesn’t work well anymore because most people make the second choice in the first problem and the first choice in the second problem, which is consistent with expected utility

Risk Preferences: Prospect Theory to the Rescue?

Prospect theory: two differences with expected utility theory

- probability distortion: people tend to exaggerate low probabilities
- reference point: people are not concerned with overall well-being but gains and losses relative to a reference point

the reference point is vague: treated as an unknown value changing from setting to setting in an unexplained manner

a bug that renders the theory unusable for economists

Probability Weighting

Bruhin, Fehr-Duda, and Epper [2007] (economists) estimate probability weighting functino in the laboratory

p_i is the chance of winning one of two prizes $x_i \geq 0$ where $i = 1{,}2$

they conclude that the single largest group of people use a utility function

$$U = \sum_i \frac{.846 p_i^{.414}}{.846 p_i^{.414} + (1 - p_i)^{.414}} x_i^{1.056}$$

Note non-linearity in probabilities

What Else Does This Theory Predict?

A. $5,000 for sure (or)

B. a 50-50 coin-flip between $9,700 dollars and nothing

- People in the large group using the utility function above should pick B
- Predicts no Allais paradox

Loss Domain

one of the main hypotheses in prospect theory: risk averse for gains, but risk loving for losses

- hard to present people with the possibility of losses in the laboratory
- initial stake: can change their "reference point"
- consequence: experiments with gains typically for larger stakes than losses
- most of the losses hypothetical rather than real
- maybe people are risk loving for small (real) stakes and risk averse for (real) large stakes?

Bosch-Domenech and Silvestre 2006

Clever design:

- endowed subjects with money in one experiment
- conducted the gambles in a second experiment several months later
- what they found: risk aversion and risk loving are driven by stakes and not by losses and gains.

Risk Loving for Losses in the Field

in the recent crisis (and historically) bankers willing to gamble a small probability of a large loss for a modest increase in the average return

Godlewski 2007 argues that this is predicted by prospect theory

- However: standard expected utility theory predicts bankers should gamble on losses.
- the “hail Mary pass” in football: if you are behind gamble, only winning counts, not how much you lose by
- bankers face similar situation: if they win they keep the money; if they lose the government bails them out

Subjective Uncertainty and the Ellsburg Paradox

Ellsberg [1961]

Two urns: each contains red balls and black balls

Urn 1: 100 balls, how many red or black is unknown

Urn 2: 50 red and 50 black

Choice 1: bet on urn 1 red or urn 2 red

Choice 2: bet on urn 1 black or urn 2 black

Urn 1: 100 balls, how many red or black is unknown

Urn 2: 50 red and 50 black

Choice 1: bet on urn 1 red or urn 2 red **[urn 2]**

Choice 2: bet on urn 1 black or urn 2 black **[urn 2]**

1 says that urn 2 red more likely than urn 1 red

2 says that urn 2 black more likely than urn 2 black

but this is inconsistent with probabilities that add up to 1

Can introduce theory of “ambiguity aversion” as in Schmeidler [1989], Ghirardato and Marinacci [2000]

- Basically probabilities do not add up to one; remaining probability is assigned to “nature” moving after you make a choice and choosing the worst possibility for you. [The stock market always tumbles right after I buy stocks.]
- Note: non-economists talk about risk versus uncertainty – no model from psychology, only mainstream economists have models about this
- Distinction between risk and uncertainty is due to the economist Frank Knight – in 1921

Ellsburg Paradox Paradox

we should be able to break the indifference

Urn 1: 1000 balls, how many red or black is unknown

Urn 2: 501 red and 499 black

Choice 1: bet on urn 1 red or urn 2 black **[urn 2]**

Choice 2: bet on urn 1 black or urn 2 black **[urn 2]**

Combine this into a single choice:

Bet on urn 1 red, urn 1 black or urn 2 black

Ambiguity aversion says go with urn 2 black…

But this is a bad idea: flip a coin to decide between urn 1 red and urn 1 black

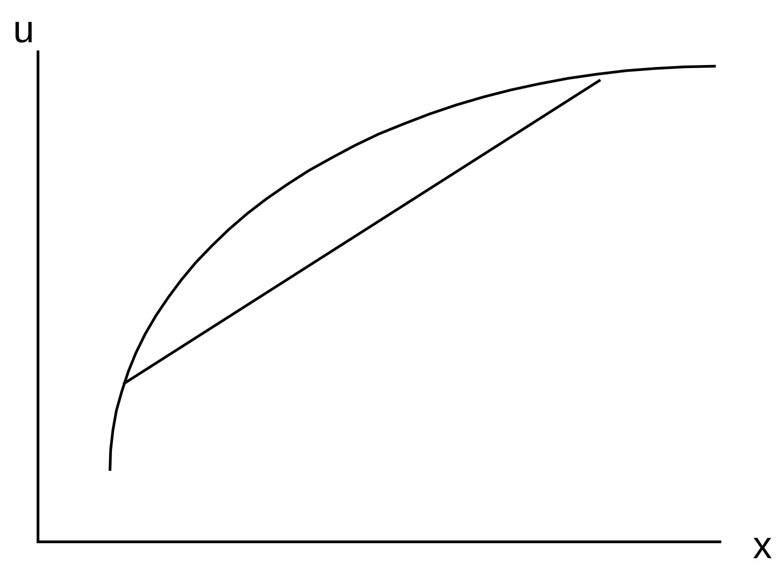
Risk Aversion

Jensen's inequality

u is a concave function if and only if $u(Ex) \geq Eu(x)$

that is: you prefer the certainty equivalent

so concavity = risk aversion



Risk premium

y a random income with $Ey = 0, Ey^2 = 1$

$u(x - p) = Eu(x + \sigma y)$

Taylor series expansion:

$$u(x) - pu'(x) = E\left[u(x) + \sigma u'(x)y + (1/2)\sigma^2 u''(x)y^2\right]$$
$$= u(x) + (1/2)\sigma^2 u''(x)$$

so $p = -\dfrac{u''(x)}{u'(x)}\dfrac{\sigma^2}{2}$

we can also consider the relative risk premium

$u(x - \rho x) = Eu(x + \sigma y x)$

$$\rho = -\frac{u''(x)x}{u'(x)}\frac{\sigma^2}{2}$$

Measures of Risk Aversion

Absolute risk aversion

The coefficient of absolute risk aversion is $-\frac{u''(x)}{u'(x)}$

Relative risk aversion

The coefficient of relative risk aversion is $-\frac{u''(x)x}{u'(x)}$

Changes in Risk Aversion with Wealth

We ordinarily think of absolute risk aversion as declining with wealth (this is a condition on the third derivative of u).

Constant relative risk aversion

$u(x) = \frac{x^{1-\rho}}{1-\rho}$ also known as "constant elasticity of substitution" or CES

$\rho \geq 0$

$$-\frac{u''(x)x}{u'(x)} = \frac{\rho x^{-\rho-1}x}{x^{-\rho}} = \rho$$

$\rho = 0$ linear, risk neutral

$\rho = 1 \;\; u(x) = \log(x)$

useful for empirical work and growth theory

note that constant relative risk aversion implies declining absolute risk aversion

How risk averse are people?

Equity premium

Mehra and Prescott [1985]; Shiller [1989] data annual 1871-1984

Mean real return on bonds r_b=1.9%; Mean real return on S&P 7.5%

Equity premium $\bar{\lambda} = .056$

Standard error of real stock return 18.1%, $\sigma = 0.181$.

normalized real per capita consumption standard error $s = .035$

let x denote initial wealth

Let α be fraction of portfolio in S&P

calculate consumption

$$u((1-\alpha)x(1+r_b)+\alpha x(1+\bar{r}_s+\sigma y))=$$

$$u(x+xr_b+\alpha\bar{\lambda}x+\alpha\sigma yx)$$

$$\frac{d}{d\alpha}Eu(x+xr_b+\alpha\bar{\lambda}x+\alpha\sigma yx)$$

$$=\bar{\lambda}xEu'+\sigma xEyu'$$

$$=u'\bar{\lambda}x+\bar{\lambda}xEu''()[\alpha\sigma yx]+\sigma xEyu'(x+xr_b+\alpha\bar{\lambda}x+0)+\sigma xEyu''()[\alpha\sigma yx]$$

$$=u'\bar{\lambda}x+\alpha u''\sigma^2x^2=0$$

$$\rho=\bar{\lambda}/(\alpha\sigma^2)\approx 1.81\alpha^{-1}$$

$$s^2=\mathrm{var}[((1-\alpha)x+\alpha(1+\bar{\lambda})x+\alpha\sigma yx)/x]=\alpha^2\sigma^2$$

or $\alpha^{-1}\approx\sigma/s=5.17$ giving $\rho=8.84$

Risk Aversion in the Laboratory

In laboratory experiments we often observe what appears to be risk averse behavior over small amount of money (typical payment rates are less than $50/hour, and play rarely lasts two hours)

How can people be risk averse over gambles involving such an insignificant fraction of wealth?

Rabin [2000]: Risk aversion in the small leads to impossible results in the large

“Suppose we knew a risk-averse person turns down 50-50 lose $100/gain $105 bets for any lifetime wealth level less than $350,000, but knew nothing about the degree of her risk aversion for wealth levels above $350,000. Then we know that from an initial wealth level of $340,000 the person will turn down a 50-50 bet of losing $4,000 and gaining $635,670.”

Here we need a reference point

Intertemporal Preference: Additive Separability and Impatience

discounted utility

$\sum_{t=1}^{T} \delta^{t-1} u_t$

infinite discounted utility

$\sum_{t=1}^{\infty} \delta^{t-1} u_t$

average discounted utility

$(1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} u_t$

note that average present value of 1 unit of utility per period is 1

The real equity premium puzzle

Utility $u(x) = \frac{x^{1-\rho}}{1-\rho}$, $\sum_{t=1}^{\infty} \delta^{t-1} u_t$

Consumption grows at a constant rate $x_t = \gamma^t$

$u'(x) = x^{-\rho}$

marginal rate of substitution $\frac{1}{1+r} = \frac{\delta u'(x_{t+1})}{u'(x_t)} = \frac{\delta \gamma^{-\rho(t+1)}}{\gamma^{-\rho t}} = \delta \gamma^{-\rho}$

1889-1984 from Shiller [1989]

average real US per capita consumption growth rate 1.8%

$\rho = 8.84 \;\; r = 17\%$

Mean real return on bonds 1.9%; Mean real return on S&P 7.5%

http://www.dklevine.com/econ201/interest.xls

How does the market react to good news?

Value of claims to future consumption relative to current consumption

$x_1 = 1$

$$\frac{\sum_{t=2}^{\infty} \delta^{t-1} u'(x_t) x_t}{u'(1)}$$

$$\sum_{t=2}^{\infty} \delta^{t-1} \gamma^{-(t-1)\rho} \gamma^{t-1} = \sum_{t=1}^{\infty} \left[\delta\gamma^{1-\rho}\right]^t = \frac{\delta\gamma^{1-\rho}}{1-\delta\gamma^{1-\rho}}$$

to be finite we need $\delta\gamma^{-\rho} < 1$

$$\frac{\partial}{\partial\gamma} \frac{\delta\gamma^{1-\rho}}{1-\delta\gamma^{1-\rho}} = \frac{\delta(1-\rho)\gamma^{-\rho}}{(1-[\delta\gamma^{-\rho}])^2}$$

$\rho > 1$ this is negative

Present Bias

Fraction of People Making Choice

Scenario	Choices	Fraction Making Choice
1	$175 now	0.82
	$192 in 4 weeks	0.18
2	$175 in 26 weeks	0.37
	$192 in 30 weeks	0.63

Keren and Roelsofsma [1995] (60 observations)

Quasi-hyperbolic Discounting

$$u(c_1) + \theta \sum_{t=2}^{\infty} \delta^{t-1} u(c_t)$$

studied by economist Richard Strotz in the mainstream *Review of Economic Studies* in 1955

Laibson [1997] examines consequences for consumption: in the mainstream *Quarterly Journal of Economics* – cited 1543 times, so not overlooked by economists

Hyperbolic/quasi-hyperbolic discounting predicts that present bias should not depend on whether or not the reward is uncertain

fraction Making Choice With Uncertain Reward

Scenario	Choices	Probability of reward	
Scenario		1.0 (60)	0.5 (100)
1	$175 now	0.82	0.39
	$192 in 4 weeks	0.18	0.61
2	$175 in 26 weeks	0.37	0.33
	$192 in 30 weeks	0.63	0.67

Preference Uncertainty

(based on Villaverde and Mukherji [2003])

Uncertainty about preferences 100 days from now.

- marginal utility of consumption can take on two values 1 or 2 with equal probability
- daily subjective discount factor to a good approximation 1.
- value of todays and tomorrow's marginal utility is know with certainty
- offered 10 today versus 15 tomorrow, you take 10 today with probability .25.
- preferences 100 days from now are unknown
- ratio of expected utilities is 1: if you are offered 10 in 100 days versus 14 in 101 days you always take 14

Commitment?

1) $175 in twenty six weeks

2) $192 in thirty weeks

3) wait and make the decision twenty six weeks from

single rational decision maker would always prefer flexibility of waiting

self-commitment – intentionally limiting our future options – not consistent with standard economic models

- Schelling example

Della Vigna and Malemendier 2006 health club memberships

- long-term memberships paid on average $17 per visit as against a $10 per visit fee
- ignore hassle factor of availability of lockers and the need to pay each visit: some evidence that people are trying to make a commitment to attending the health club

Self-Control Models

2001: Faruk Gul and Wolfgang Pesendorfer; Dekel, Lipman and Rustichini

preferences over menus – lists of which options will be available – with preference for commitment

major topic of ongoing research

- behavioral: postulate rather than a single decision maker each of us has several conflicting selves and that the resolution of internal conflict leads to our final decisions
- pioneered by behaviorists: Shefrin and Thaler in 1981 and Ainslie in 2001
- obvious role in explaining things such as impulsive behavior and drug addiction

A Note of Caution

Supposed spur of the moment, giving in to temptation and taking immediate gratification at the expense of the future often not

Gambling and sexual behavior

- Eliot Spitzer lost his job as governor of New York because of his "impulsive" behavior in visiting prostitutes
- In fact he paid months in advance (committing himself to seeing prostitutes rather than committing himself to avoiding them)
- once flew a prostitute from Washington D.C. to New York – violating Federal as well as State law
- Rush Limbaugh carrying large quantities of viagra from the Dominican Republic: suspected that he had gone there on a "sex vacation"
- Not done impulsively at the last minute

the Las Vegas vacation? planned well in advance

Procrastination

prominent on Akerlof's list of "behavioral" phenomenon

I quizzed a behavioral economist for examples

- Paying taxes the day before the deadline
- Christmas shopping on Christmas eve
- Buying party supplies for something like a New Years Eve party or a 4th of July party at the last minute
- Buying Halloween costumes at the last minute
- Delaying the purchase of concert tickets
- Waiting to buy plane tickets for Thanksgiving

Irrationality?

an unpleasant task is delayed until the deadline

if task is unpleasant and we are impatient: the best thing to do is to wait until the deadline

The Horse and the King

The king had a favorite horse that he loved very much. It was a beautiful and very smart stallion, and the king had taught it all kinds of tricks. The king would ride the horse almost every day, and frequently parade it and show off its tricks to his guards.

A prisoner who was scheduled to be executed soon saw the king with his horse through his cell window and decided to send the king a message. The message said, “Your Royal Highness, if you will spare my life, and let me spend an hour each day with your favorite horse for a year, I will teach your horse to sing.”

The king was amused by the offer and granted the request. So, each day the prisoner would be taken from his cell to the horse's paddock, and he would sing to the horse “La-la-la-la” and would feed the horse sugar and carrots and oats, and the horse would neigh. And, all the guards would laugh at him for being so foolish.

One day, one of the guards, who had become somewhat friendly with the prisoner, asked him, “Why do you do such a foolish thing every day singing to the horse, and letting everyone laugh at you?

You know you can't teach a horse to sing. The year will pass, the horse will not sing, and the king will execute you.”

The prisoner replied, “A year is a long time. Anything can happen. In a year the king may die. Or I may die. Or the horse may die. Or ...The horse may learn to sing.”

The Health Club Again

evidence that people pay extra to self-commit to exercising

procrastination: delay canceling memberships after stopping attendance

average of 2.3 months before canceling

average amount lost is nearly $70

no issue of delaying an unpleasant task until a deadline

irrational procrastination?

Analysis

may take a while after last attending to make the final decision to quit

still: why cancel now when we could cancel next month instead?

Are people *naïve*: do not understand that they are procrastinators?

they put off until tomorrow, believing they will act tomorrow, and do not understand that tomorrow they will face the same problem and put off again

maybe some people that behave this way

several kinds of untrue beliefs that might lead to procrastination

in the face of a lifetime of evidence people may falsely believe that they are not procrastinators

An Alternative from Learning Theory

Della Vigna and Malemendier assert that a canceling a membership is a simple inexpensive procedure: suppose true

- people might falsely believe that it will be a time consuming hassle
- they foolishly they think canceling will involve endless telephone menus, employees who vanish in back rooms for long periods of times and so forth
- are people more likely to be misinformed about something they have observed every day for their entire lives (whether or not they are procrastinators) or something that they have observed infrequently and for which the data indicates costs may be high (canceling)?
- learning theory suggests the latter