Economics 201B: Game Theory Problem Set (#2)

The Chain Store Paradox Paradox

Consider the Kreps-Wilson version of the chain store paradox: An entrant may stay out and get nothing (0), or he may enter. If he enters, the incumbent may fight or acquiesce. The entrant gets *b* if the incumbent acquiesces, and $b-1$ if he fights, where $0<b<1$. There are two types of incumbent, both receiving $a>1$ if there is no entry. If there is a fight, the strong incumbent gets 0 and the weak incumbent gets -1; if a strong incumbent acquiesces he gets -1, a weak incumbent 0.

Only the incumbent knows whether he is weak or strong; it is common knowledge that the entrant a priori believes that he has a π_0 chance of facing a strong incumbent. Define

$$\gamma = \frac{p_0}{1-p_0}\frac{1-b}{b}$$

a. Sketch the extensive form of this game.

b. Define a sequential equilibrium of this game.

c. Show that if $\gamma \neq 1$, there is a unique sequential equilibrium, and that if $\gamma > 1$ entry never occurs, while if $\gamma < 1$ entry always occurs.

d. What are the sequential equilibria if $\gamma = 1$?

e. Now suppose that the incumbent plays a second round against a different entrant who knows the result of the first round. The incumbent's goal is to maximize the sum of his payoffs in the two rounds. Show that if $\gamma > 1$ there is a sequential equilibrium in which the entrant enters on the first round and both types of incumbents acquiesce. Be careful to specify both the equilibrium strategies and beliefs.

Courtroom Drama

Two players: plaintiff and defendant, in a civil suit. The plaintiff knows whether or not he will win the case if it goes to trial, but the defendant does not. The defendant's beliefs are $\Pr(\text{plantiff wins}) = 1/3$. This is common knowledge.

The cost of the trial is 1. The loser of the trial bears this cost. If the plaintiff wins the trial, then the defendant will have to pay the plaintiff 3 and also pay for the cost of the trial. If the plaintiff loses, he'll have to pay for the cost of the trial, and the defendant will neither win anything nor lose anything.

The plaintiff has two actions: ask for a low settlement, m = 1, or ask for a high settlement, m = 2. If the defendant accepts m, then the defendant is agreeing to pay m to the plaintiff out of court. If the defendant rejects m, the case goes to court.

(a) Draw the game tree.

(b) Find all sequential equilibria.

Education and Employment

There are two players: a worker (player 1) and a firm (player 2). The worker has two types: $\overline{\theta}, \underline{\theta}$, where $0 < \underline{\theta} < \overline{\theta}$ and $\Pr(\overline{\theta}) = p$. The worker knows his type and chooses whether or not to get an education, at a cost $c(\theta)$. The firm observes the worker's education decision. The firm then offers a wage: $\overline{w}, \underline{w}$. Finally, the worker accepts or rejects the wage. The profit to the firm from employing the worker is $\theta - w$, that is, education does not affect the worker's productivity. Suppose that $\overline{\theta} - \overline{w} > 0, \underline{\theta} - \overline{w} < 0$.

(a) Draw the game tree.

(b) For what values of p do we get pooling, separating, and semi-separating Nash equilibria?

Decreasing Absolute Risk Aversion

A continuum of consumers has utility function $u(x) = 78x - x^2$. Each consumer has a 50% chance of getting $x = 30$ and a 50% chance of $x = 10$. Consider the following "mechanism:" a consumer that announces he has $x = 30$ pays τ. A consumer that announces $x = 10$ receives a lottery with a 50% chance of winning g and a 50% chance of winning b where $.5g + .5b = \tau$. Suppose that "rich" consumers ($x = 30$) can lie and say that they are poor $(x = 10)$. Find the mechanism that maximizes the expected utility of a consumer before he knows his type, subject to the constraint that the rich consumer does not wish to lie

Moral Hazard

There are 2 states of the world $s = 1,2$ and 2 possible actions $a = 1,2$. A risk neutral principal observes only the state and not the action of the agent he hires. The net gain of an agent if he is paid w and takes action a is $v(w) - c(a)$, where $c(1) < c(2)$. Under action a the probability of state s is $p_s(a)$, where $p_2(a)$ is increasing in a. The agent's reservation utility is 0. The output (received by the principal) is y_s where $y_2 > y_1$.

(a) The principal wishes to induce action $a = 2$ and only "downward" constraints of pretending lower cost are potentially binding. What condition is sufficient for the optimal incentive scheme w_1, w_2 to be monotonic? Prove your claim.

(b) Suppose $v(w) = 1 - e^{-\gamma w}$. What more can be said about $w_2 - w_1$?

(c) For this special case, discuss the effect on the optimal incentive scheme if there is a change in the agent's reservation utility, assuming the principal still wants to induce the action $a = 2$.

(d) Is there a change in the agents reservation utility that would lead the principal to prefer to induce the action $a = 1$?

(e) Suppose that the agent's utility is $u(w, a)$ and is not separable. Is it possible to induce the agent to use $a = 2$ for arbitrarily large reservation utilities?

Adverse Selection

Consider a continuum of ex ante identical individuals with utility function for consumption *c* of $-e^{-c}$. Ex post, two states are possible. In state 1 the endowment is 2. In state 2 the endowment is 0. What is the first best allocation? Suppose that the state is privately known. Show that there is no incentive compatible ex ante exclusive contract that gives the low endowment type more utility than at autarky.