Notes on Correlated Equilibrium
Course Econ 201B

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1 How do we check that something is a Correlated equilibrium?

We have to check for all players that they do not wish to deviate from the recommended play. In doing so, we only need to consider deviations to pure strategies because there necessarily has to exist a profitable deviation to a pure strategy for a profitable deviation to a mixed strategy to exist.

2 Example

Consider the following game:

	L	R
U	2,1	-1,-1
D	0,0	1,2

Nash Equilibria are (U,L), (D,R) and $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$.
Payoffs in these equilibria are $(2, 1)$, $(1, 2)$, $\left(\frac{1}{2}, \frac{1}{2}\right)$.

Let's consider a correlating device:

	L	R
U	$\frac{1}{3}$	0
D	$\frac{1}{3}$	$\frac{1}{3}$

Check that this is a correlated equilibrium:

$u_1\left(U|U\right) = 2$
$u_1\left(D|U\right) = 0$

$u_1\left(U|D\right) = 2 \times \frac{1}{2} - 1 \times \frac{1}{2} = \frac{1}{2}$
$u_1\left(D|D\right) = 0 \times \frac{1}{2} + 1 \times \frac{1}{2} = \frac{1}{2}$

Therefore, following the recommendation is worthwile for player 1

$u_2\left(L|L\right) = 1 \times \frac{1}{2} + 0 \times \frac{1}{2} = \frac{1}{2}$
$u_2\left(R|L\right) = -1 \times \frac{1}{2} + 2 \times \frac{1}{2} = \frac{1}{2}$

$u_2\left(L|R\right) = 0$
$u_2\left(R|R\right) = 2$

Therefore, following the recommendation is worthwile for player 2
Expected payoff for player 1: $\frac{1}{3} \times 2 + \frac{1}{3} \times 0 + \frac{1}{3} \times 1 = 1$
Expected payoff for player 2: $\frac{1}{3} \times 2 + \frac{1}{3} \times 0 + \frac{1}{3} \times 1 = 1$

Is there a correlated equilibrium that puts possible probability on the negative payoffs? Yes, for example:

	L	R
U	$\frac{1}{4}$	$\frac{1}{4}$
D	$\frac{1}{4}$	$\frac{1}{4}$

It is easy to check that this is a correlated equilibrium, but it is also not necessary since it is equivalent to the mixed strategy equilibium.