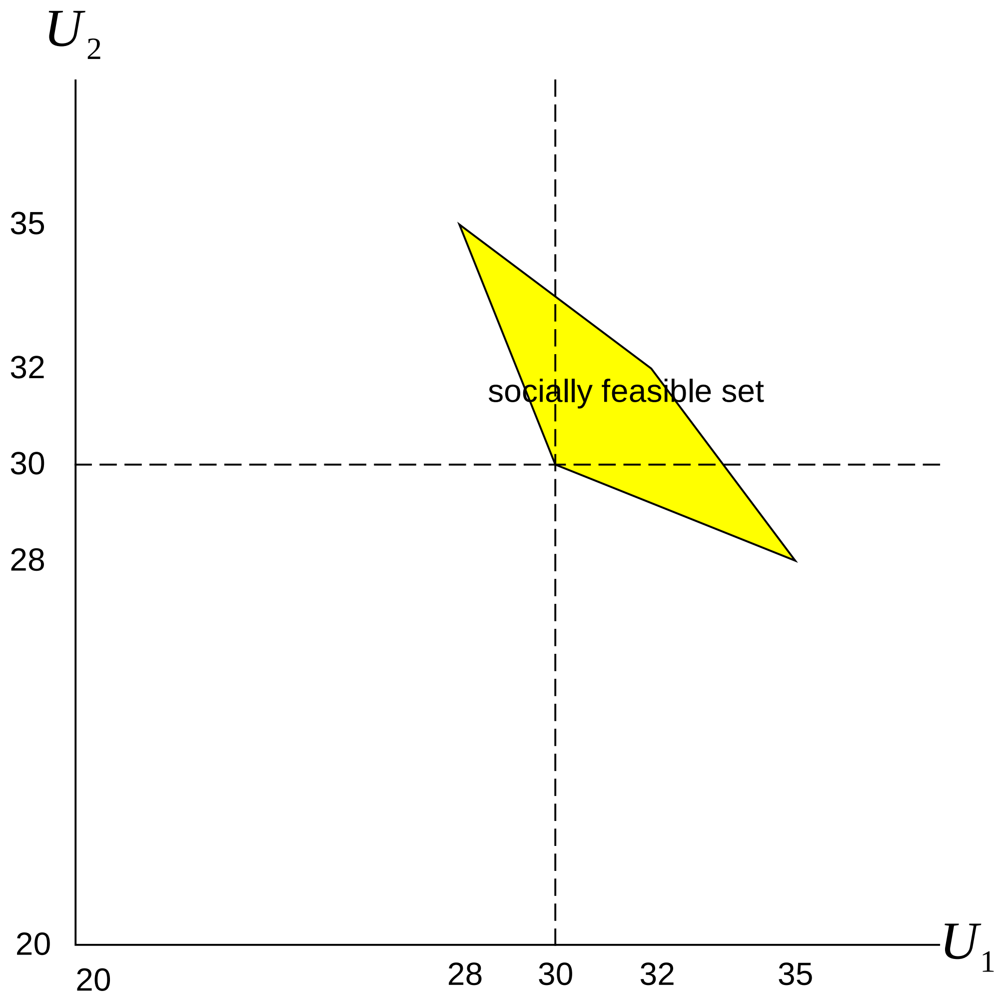
Repeated Games: Long-Run Players and the Folk Theorem

Folk Theorems

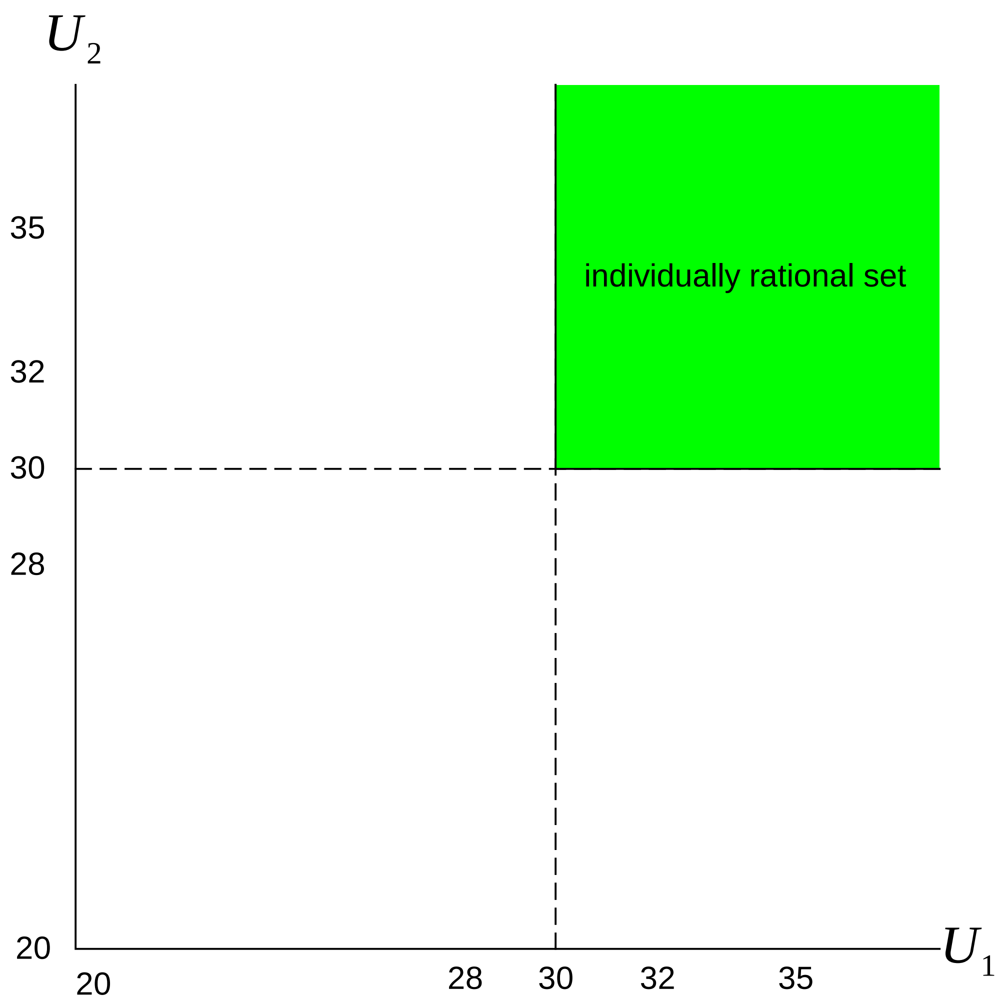
- socially feasible
- individually rational

Statement of Folk Theorem

	Player 2	
Player 1	don’t confess	confess
don’t confess	32,32	28,35
confess	35,28	30,30



U_2
35
32
30
28
20
socially feasible set
U_1
20
28
30
32
35


U_2
35
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individually rational set
U_1
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32
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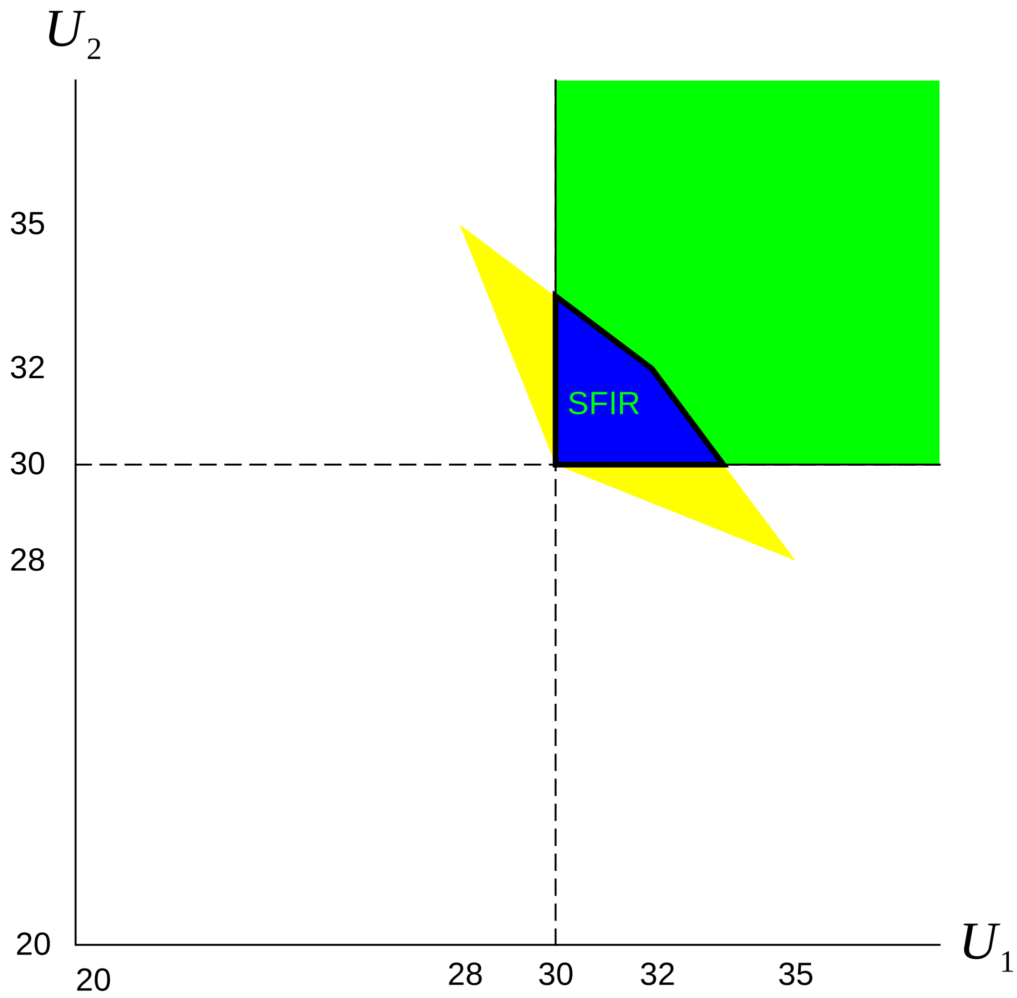
U_2
35
32
30
28
20
SFIR
U_1
20
28
30
32
35

- Nash with time averaging
- perfect Nash threats with discounting
- Fudenberg and Maskin [1986]
- Something like full dimensionality needed: why?

The Downside of the Folk Theorem

4,4	1,1
1,1	0,0

$\delta = 3/4$

D in first period

If DD in first period UU forever after

Else start over

In equilibrium get $(1/4)0 + (3/4)4 = 3$

Deviation get $(1/4)1 + (3/4)3 = 10/4 = 2.5$

In general want $(1-\delta)0+\delta 4=\delta 4 \geq (1-\delta)1+\delta^2 4$

Or

$0 \geq \delta^2 4 - 5\delta + 1$

$$\delta = \frac{5 \pm \sqrt{25-4}}{2} = \frac{5-\sqrt{21}}{2} \approx .2087$$

For δ close to 1 the worst equilibrium is near 1 for both players

Tit-for-tat

Play the same thing that your opponent did in the previous period, cooperate in the first period

3,3	0,4
4,0	1,1

If your opponent is playing tit-for-tat, use dynamic programming

Four markov strategies:

Do the same as opponent: 3

Do opposite of opponent: $\frac{1-\delta}{1-\delta^2}4=\frac{4}{1+\delta}$ (=3 at $\delta=1/3$)

Always cooperate: 3

Always cheat: $(1-\delta)4+\delta 1=4-3\delta$ (=3 at $\delta=1/3$)

So tit-for-tat an equilibrium for $\delta\geq 1/3$

Matching and Information Systems

Juvenal in the first century A.D.

“Sed quis custodiet ipsos custodes?”

translation: “Who shall guard the guardians?”

answer: they shall guard each other.

Contagion Equilibrium

players randomly matched in a population; observe only opponent's current play

Ellison [1993]: could have cooperation due to contagion effects

3,3	0,4
4,0	1,1

Strategy: cooperate as long as everyone you have ever met cooperated; if you have ever met a cheater, then cheat

With these strategies the number of cheaters is a Markov chain with two aborbing states: all cheat, none cheat

Playing the proposed equilibrium strategy results in non cheat and a utility of 3; deviating results eventually in all cheat; this aborbing state is approached exponentially fast; the amount of time depends on the population size, but not the discount factor, so for discount factor close enough to one it is optimal not to cheat

But contagion effects diminish as population size grows, and the equilibrium is not robust to noise, which will trigger a collapse

Information Systems-Example

Overlapping generations; young matched against old:

Only the young have a move – give a gift to old person

Gift worth $x > 1$ to old person; costs 1 to give the gift

Information system: assigns a young person a flag based on their action and the old person’s flag

Consider the following information system and strategies:

Cooperate against a green flag -> green flag

Cheat against a red flag -> green flag

On the other hand

Cheat against green flag -> red flag

Cooperate against red flag -> red flag

If you meet a green flag:

Cooperate you get $x - 1$

Cheat you get 0

If you meet a red flag

Cheat you get x

Cooperate you get -1

So it is in fact optimal to cooperate against green (your team) and cheat against red (the other team)

Notice that this is a **strict** Nash equilibrium if there is noise (so that there are some red flags)

Notice that always cheat no matter what the flags is also a strict Nash equilibrium

Information Systems-Folk Theorem

Kandori [1992]

$u^i(a)$

I a finite set of information states

$\eta : A \times I^2 \to I$ an information system

if at *t* you and your opponent played a_t and had states η_t^i, η_t^{-i}, then your next state is $\eta_{t+1}^i = \eta(a_t, \eta_t^i, \eta_t^{-i})$

players randomly matched in a population

observe their current opponents current state

Folk Theorem for information systems: socially feasible individually rational payoff – exists an information system that supports it

Example

Prisoner's dilemma

	C	D
C	x, x	$0, x+1$
D	$x+1, 0$	1,1

$I = \{r, g\}$

$$\eta(a^i, \eta^{-i}) = \begin{cases} G & (a^i, \eta^{-i}) = C, G \\ R & (a^i, \eta^{-i}) = C, R \\ R & (a^i, \eta^{-i}) = D, G \\ G & (a^i, \eta^{-i}) = D, R \end{cases}$$

"green team strategy"

defect on red

cooperate on green

$$V(g) = x$$
$$V(r) = \delta x$$

C $(1-\delta)x + \delta V(g) = x$

D $(1-\delta)(x+1) + \delta V(r) = (1-\delta)(x+1) + \delta^2 x =$
$(1-\delta) + (1-\delta+\delta^2)x$

$$x \geq (1-\delta) + (1-\delta+\delta^2)x$$

So $\delta(1-\delta)x \geq (1-\delta)$

$$\delta \geq 1/x$$

More Versions

Folk theorem for stochastic games: Dutta, P. (1995): “A Folk Theorem for Stochastic Games,” *Journal of Economic Theory*

- Long run payoff possibilities approximately independent of current state

Finite folk theorem: Benoit, J-P. and V. Krishna (1985): “Finitely Repeated Games,” *Econometrica* **53**: 905-922

- If you have multiple Nash equilibria in the stage game

Self Referential Games

Symmetric game with finite action spaces A

Payoffs $U(a,a')$.

S finite set of strategies

Y finite set of signals

if $s \in S$ then $s : Y \to A$ (all maps $Y \to A$ are represented)

private signal y received with probability $\pi(y \mid s,s')$

what you can learn about your opponent's intentions prior to interaction

(poker players study each other's faces to see if the other is bluffing)

Perfect Identification

Cannot have $Y = S$ (why not?)

Suppose:

a $y_0 \in Y$ such that $\pi(y_0 \mid s, s) = 1$ for every $s \in S$ and $\pi(y_0 \mid s, s') = 0$ for $s \neq s'$

Can tell if the opponent has the same strategy as you

Pure minmax:

$\max_{a' \in A} U(a', a'') \geq \max_{a' \in A} U(a', a_*) \equiv u_*, \forall a'' \in A$

Folk Theorem: If $v = U(a, a)$ for some $a \in A$ and $v \geq u_*$ then v is a Nash equilibrium payoff.

Proof:

$$s_{aa*}(y) = \begin{cases} a & \text{if } y = y_0 \\ a* & \text{if } y \neq y_0 \end{cases}$$