Repeated Games

Long-Run versus Short-Run Player

a fixed simultaneous move *stage game*

Player 1 is long-run with discount factor δ

actions $a^1 \in A^1$ a finite set

utility $u^1(a^1, a^2)$

Player 2 is short-run with discount factor 0

actions $a^2 \in A^2$ a finite set

utility $u^2(a^1, a^2)$

the "short-run" player may be viewed as a kind of "representative" of many "small" long-run players

Repeated Game

history $h_t = (a_1, a_2, \ldots, a_t)$

null history h_0

behavior strategies $\alpha_t^i = \sigma^i(h_{t-1})$

Equilibrium

Nash: usual definition

Subgame perfect: usual definition, Nash after each history

Observation: the repeated static equilibrium of the stage game is a subgame perfect equilibrium of the finitely or infinitely repeated game

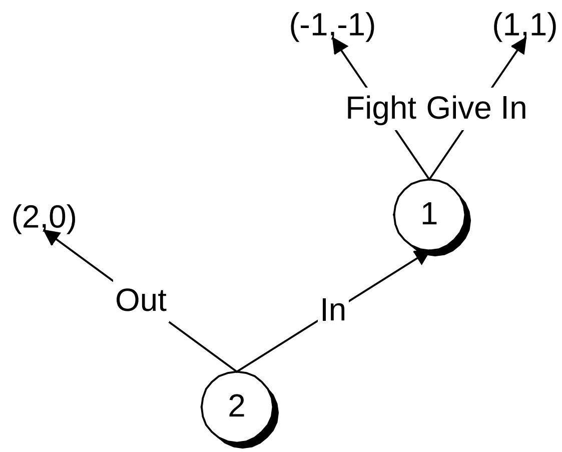
strategies: play the static equilibrium strategy no matter what

"perfect equilibrium with public randomization"

may use a public randomization device at the beginning of each period to pick an equilibrium

key implication: set of equilibrium payoffs is convex

Example: chain store game



normal form

	out	in
fight	2,0*	-1,-1
give in	2,0	1,1**

Nash

subgame perfect is In,Give In

variation on chain store

	out	in
fight	2-ε , 0	-1,-1
give in	2,0	1,1**

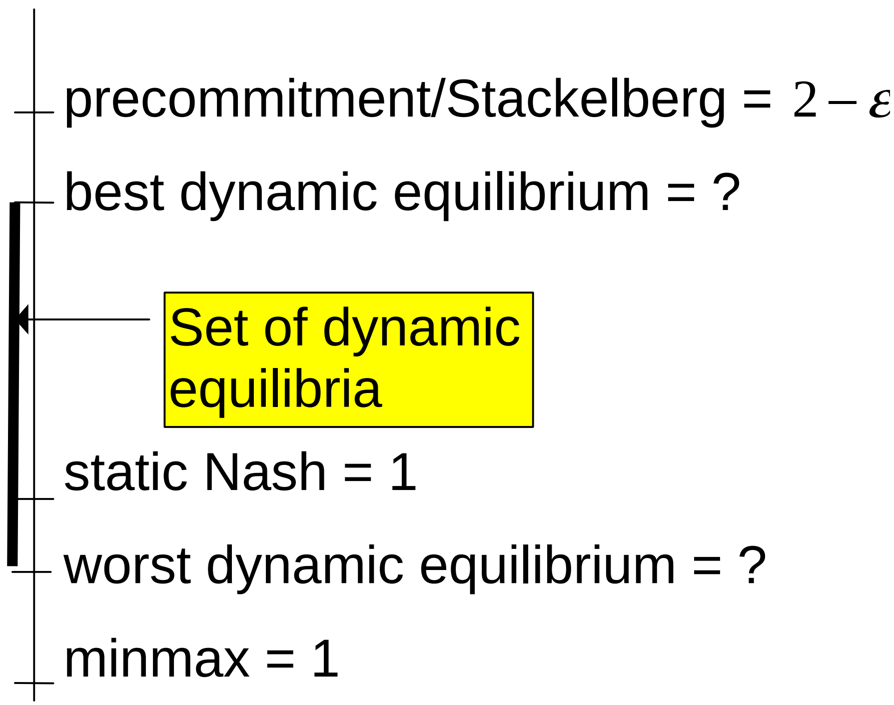
now the only equilibrium is In, Give In

payoff at static Nash equilibrium to LR player: 1

precommitment or Stackelberg equilibrium

precommit to fight get $2-\varepsilon$

minmax payoff to LR player: 1 by giving in utility to long-run player



Repeated Chain Store

finitely repeated game

final period: In, Give, so in every period

Do you believe this??

Infinitely repeated game

begin by playing Out, Fight

if Fight has been played in every previous period then play Out, Fight

if Fight was not played in a previous period play In, Give In (reversion to static Nash)

claim: this is subgame perfect

clearly a Nash equilibrium following a history with Give In

SR play is clearly optimal

for LR player

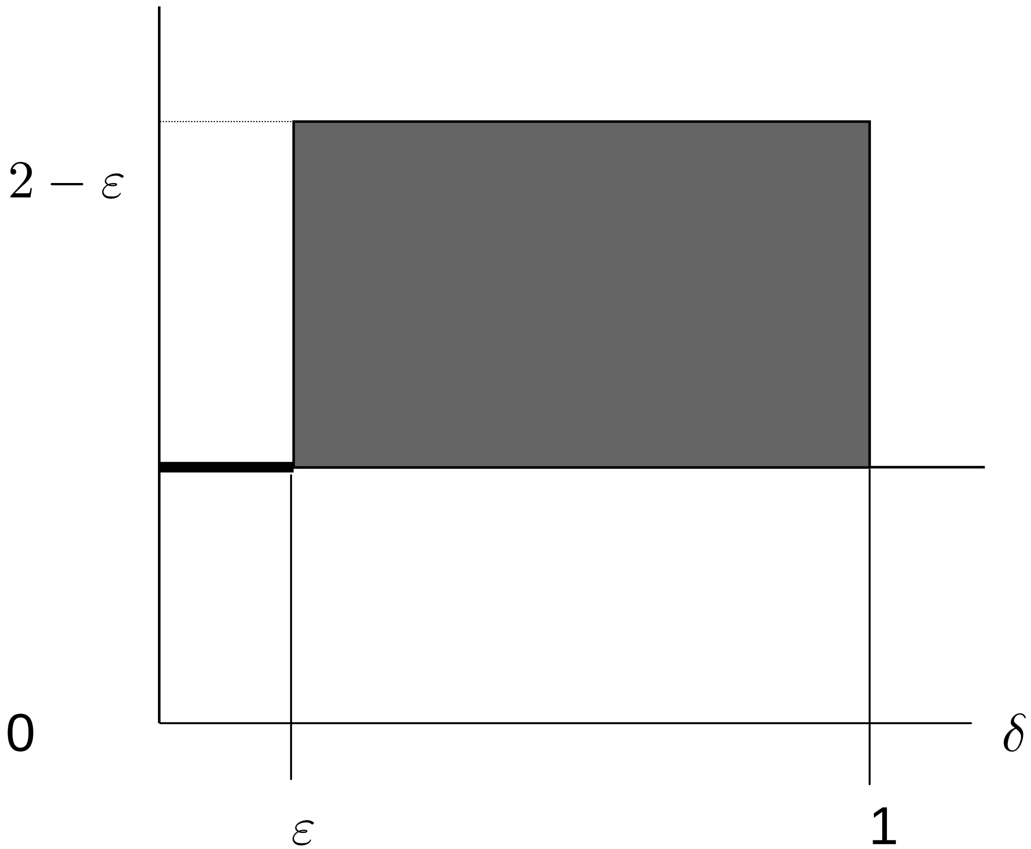
may Fight and get $2 - \varepsilon$

or give in and get $(1 - \delta)2 + \delta 1$

so condition for subgame perfection

$$2 - \varepsilon \geq (1 - \delta)2 + \delta 1$$
$$\delta \geq \varepsilon$$

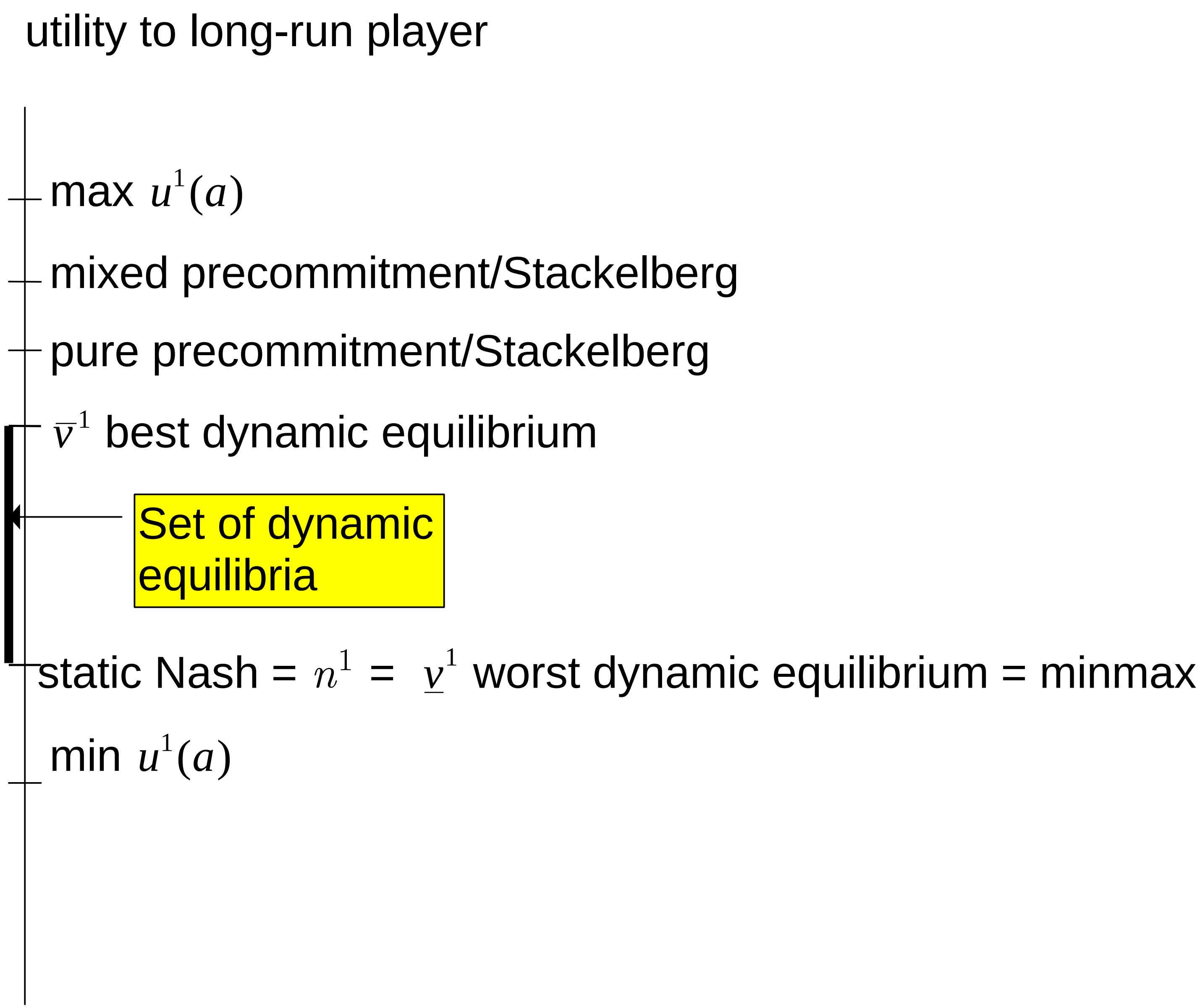
equilibrium utility for LR



General Deterministic Case

Fudenberg, Kreps and Maskin [1990]

We'll do the special case of "exit games" where the short-run player has a non-participation option so that the worst dynamic equilibrium is the static Nash of non-participation


utility to long-run player
max $u^1(a)$
mixed precommitment/Stackelberg
pure precommitment/Stackelberg
$\bar{v}^1$ best dynamic equilibrium
Set of dynamic equilibria
static Nash = n^1 = $\underline{v}^1$ worst dynamic equilibrium = minmax
min $u^1(a)$

Characterization of Equilibrium Payoff

$\alpha = (\alpha^1, \alpha^2)$ where α^2 is a b.r. to α^1

α represent play in the first period of the equilibrium

$w^1(a^1)$ represents the equilibrium payoff beginning in the next period

$v^1 \geq (1-\delta)u^1(a^1, \alpha^2) + \delta w^1(a^1)$

$v^1 = (1-\delta)u^1(a^1, \alpha^2) + \delta w^1(a^1), \alpha^1(a^1) > 0$

$n^1 \leq w^1(a^1) \leq \overline{v}^1$

how big can $w^1(a^1)$ be in = case?

Biggest when $u^1(a^1,\alpha^1)$ is smallest, in which case

$w^1(a^1) = \overline{v}^1$

$\overline{v}^1 = (1-\delta)u^1(a^1,\alpha^2) + \delta\overline{v}^1$

conclusion for fixed α

$\min_{a^1|\alpha(a^1)>0} u^1(a^1,\alpha^2)$

that is worst in support

$\overline{v}^1 = \max_{\alpha^2 \in BR^2(\alpha^1)} \min_{a^1|\alpha(a^1)>0} u^1(a^1,\alpha^2)$

Relation to Precommitment

mixed precommitment$\geq \overline{v}^1 \geq$pure precommitment

Modified Chain Store Example

	out	in
fight	2-ε, 0	-1,-1
give in	2,0	1,1

p(fight)	BR	worst in support
1	out	$2-\varepsilon$
$\frac{1}{2}<p<1$	out	$2-\varepsilon$
$0<p<\frac{1}{2}$	in	-1
p=0	in	1

Mixed Strategies

	L	M	R
U	0,-3	1,2	0,3
D	0,3*	2,2	0,0

static Nash gives 0

minmax gives 0

worst payoff in fact is 0

pure precommitment also 0

Static Analysis

p is probability of up

to get more than 0 must get SR to play M

$-3p + (1-p)3 \leq 2$ and $3p \leq 2$

first one

$-3p + (1-p)3 \leq 2$

$-3p - 3p \leq -1$

$p \geq 1/6$

second one

$3p \leq 2$

$p \leq 2/3$

want to play D so take $p = 1/6$, get $1/6 + 10/6 = 11/6$

utility to long-run player

max $u^1(a)$=2

mixed precommitment/Stackelberg=11/16

$\bar{v}^1$ best dynamic equilibrium=1

pure precommitment/Stackelberg=0

Set of dynamic equilibria

static Nash=0 = $\underline{v}^1$ worst dynamic equilibrium = minmax = min $u^1(a)$

Calculation of best dynamic equilibrium payoff

p is probability of up

p	BR^2	worst in support
<1/6	L	0
1/6<*p*<*5/6*	M	1
p>5/6	R	0

so best dynamic payoff is 1