2016 - Problem set 1 - Instititutions

Why Did the West Extend the Franchise? Democracy, Inequality, and Growth in Historical Perspective. Acemoglu, Robinson, The Quarterly Journal of Economics, 2000

We consider an infinite horizon economy with a continuum 1 of agents. A proportion $1-\lambda$ of these agents are poor, while the remaining λ form a rich elite. Assumption: $1-\lambda > \frac{1}{2}$. $i = r, p$ describes the type of the agents, r for rich (the elite), p for poor.

Income and Preferences Each individual of group i has a constant flow of income y^i each period. We assume $y^r > y^p$ (the rich are richer than the poor). Mean income is denoted by $\bar{y}$. We parametrize inequality with θ, the share of the total income going to the rich. Hence:

$$y^p = \frac{(1-\theta)\bar{y}}{1-\lambda} \quad \text{and} \quad y^r = \frac{\theta\bar{y}}{\lambda}$$

Note that

$$y^p < y^r \Rightarrow \theta > \lambda$$

The political system determines a nonnegative tax rate $0 \leq \tau \leq 1$ proportional to income, the proceeds of which are redistributed lump sum to all citizens. Let T be the total lump-sum transfer (and also per capita as we have a measure 1 of agents). The tax technology has a cost (deadweight cost of taxation), that we capture through an aggregate cost. When the government taxes an amount y at rate τ, the aggregate loss is $C(\tau)y$. We assume the following :$C : [0,1] \to \mathbb{R}_+$, $C(0) = 0$, $C'(.) > 0$, $C''(.) > 0$.

Posttax income of type i at time t is called $\hat{y}_t^i$, where

$$\hat{y}_t^i = (1-\tau_t)y^i + T_t$$

and the budget constraint of the government yields:

$$T_t = \tau_t\left((1-\lambda)y^p + \lambda y^r\right) - C(\tau_t)\bar{y} = (\tau_t - C(\tau_t))\bar{y}$$

Agents maximize the discounted sum of post-tax incomes with discount factor $\beta \in (0,1)$, or, for type i at time $t = 0$:

$$U^i = E_0 \sum_{t=0}^{\infty} \beta^t \hat{y}_t^i$$

Question 1 Take the static framework, a one-period game. Assume the rich control the political process and set the tax rate. What is their optimal tax rate τ_r^*? Assume the poor control the political process and set the tax rate. What is their optimal tax rate τ_p^*?

Political Process and Resulting Payoffs $P \in \{E, D\}$ is the state of the political process. $P = E$ means that the elite controls the political process. $P = D$ means Democracy. At $t = 0$, $P = E$. The elite can decide to extend the franchise to everybody. If not, it sets the tax rate. If it does (extend the franchise), then $P = D$, and the tax rate is set up by the median voter. Once the franchise is extended, it is forever.

The $1-\lambda$ poor can overthrow the existing government through a revolution. If attempted, a revolution always succeeds, the poor seize the income of the elite, carry out large-scale redistribution, splitting the income equally between them. But a fraction $1-\mu$ of the income is destroyed. The value of μ is stochastically determined each period (no correlation through time):

$$\mu = \begin{cases} \mu^h > 0 & w.p. \quad q > 0 \\ \mu^l = 0 & w.p. \quad 1-q \end{cases}$$

Timing within a period

1. The state μ is revealed
2. The elite decide whether to extend the franchise. If they don't ($P = E$), they set the tax rate.
3. The poor decide whether to initiate a revolution. If they do, they share the remaining income. If they don't and if the franchise has been extended ($P = D$), the median voter sets the taxe rate.

Simplification Note that all poor agents and all elite agents are identical. We simplify the game to a dynamic 2-player game, between a rich and a poor. We thus omit the potential free-riding problem inherent to revolutions.

Part I . (Pure Strategy) Markov Perfect Equilibria (MPE)

We first focus on MPE, in which strategies only depend on the current state of the world and not on the entire history of the game.

Strategy for the elite in step 2

$$\sigma^r : \begin{array}{lcl} \{\mu^h, \mu^l\} \times \{E, D\} & \rightarrow & \{0,1\} \times [0,1] \\ (\mu, P) & \mapsto & (\phi, \tau^r) \end{array}$$

where $\phi = 0 \Rightarrow P$ stays at E, $\phi = 1 \Rightarrow$ the elite extend the franchise and P switches to D forever. τ^r is the tax rate set by the elite (only implemented if the elite is in power, $P = E$).

Strategy for the poor in step 3

$$\sigma^p : \begin{array}{lcl} \{\mu^h, \mu^l\} \times \{E, D\} \times \{0,1\} \times [0,1] & \rightarrow & \{0,1\} \times [0,1] \\ (\mu, P, \phi, \tau^r) & \mapsto & (\rho, \tau^p) \end{array}$$

where $\rho = 0$ means no revolution, $\rho = 1$ means revolution, and τ^p is the tax rate set by the poor (only implemented if $P = D$).

(Pure Strategy) MPE is the following:

$\{\sigma^r(\mu, P), \sigma^p(\mu, P|\phi, \tau^r)\}$ such that σ^p and σ^r are best-responses to each other, $\forall \mu, P$.

Question 2 Let $V^i(R, \mu), i = p, r$ the value for agent i when a revolution is started, in state μ. Compute $V^i(R, \mu)$, for $i = r, p$. Would the poor decide to initiate a revolution when $\mu = \mu^l$?

Question 3 Compute $\sigma^r(\mu^l, E)$. Let $V^i(\mu, P)$ the value for agent i of being in state μ, P at the beginning of a period. Compute $V^i(\mu^l, E), \forall i$.

Question 4 Let $\tilde{V}^i(E)$ the value for agent i of being in state E for ever, without Revolution, and having the elite always play their preferred tax rate τ_r^*. Compute it and write what we call the **"Revolution Constraint"**: (we assume that if there is indifference, the poor do not start a revolution)

$$V^p(R, \mu^h) \leq \tilde{V}^p(E) \tag{RC}$$

What is the condition on θ so that the constraint holds?

From now on, assume that (RC) *does not hold, and that without any concession form the elite, a revolution will occur.*

Question 5 Let $\tilde{V}^i(E, \mu, \tau)$ the value for agent i of being in state μ today, in state E for ever, without Revolution, and having the elite always play their preferred tax rate τ_r^* when $\mu = \mu^l$ and the tax rate τ when $\mu = \mu^h$. Compute it for $i = p, r$. Show that τ_p^* maximizes $\tilde{V}^p(E, \mu, \tau)$ (on τ) and write what we call the **"Redistribution Constraint"**:

$$V^p(R, \mu^h) \leq \tilde{V}^p(E, \mu^h, \tau_p^*) \tag{RDC}$$

Is there a τ such that $V^p(R, \mu^h) \leq \tilde{V}^p(E, \mu^h, \tau)$ and redistribution alone can prevent revolution? What is the condition on the primitives of the model such that there is none, or again (RDC) does not hold?

Question 6 Which tax rate would the median voter choose in Democracy? Let $\tilde{V}^i(D)$ the value for agent i of being in state D without Revolution, and having the median voter always play its preferred tax rate τ_p^*. Compute it for $i = p, r$ and write what we call the **"Democracy Constraint"**:

$$V^p(R, \mu^h) \leq \tilde{V}^p(D) \tag{DC}$$

If both (RDC) and (DC) are satisfied, which strategy would the Elite prefer to prevent a revolution in state μ^h?

From now on, assume that (DC) *holds, so that the poor prefer Democracy over Revolution (but Remember that* (RC) *holds, so that no concession from the elite leads to Revolution)*

Question 7 Compute $V^p(R, \mu^h)$ when $\mu^h = 0$ and when $\mu^h = 1$. Can you deduce whether there exist a $\mu^* \in (0, 1)$ such that:

$$\mu^h = \mu^* \Rightarrow \text{(RDC) holds with equality?}$$

Assume (DC) *holds, so that the poor always prefer Democracy to Revolution, but do not assume* (RC) *anymore.*

Question 8 Give, as a function of μ^h, the existence of a unique Markov Perfect Equilibrium $\{\tilde{\sigma}^r, \tilde{\sigma}^p\}$ in the repeated game, specifying what are the strategies in each part of the parameter space: $\mu^h \in [0, 1]$. You should find 3 zones for μ^h

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Part 2 . Non-Markovian Equilibria

The restriction to MPE can seem natural, because MPE highlights the commitment problem of the elite. The elite cannot commit to redistribute in future states where it is not in their immediate interest to do so (that is when $\mu = \mu^l$). But MPE rules out strategies in which the elite can make certain promises, that would be made credible by the threat that the poor would initiate a revolution once the state allows it (once $\mu = \mu^h$) as a punishment for past deviations of the elite. MPE does not allow citizens to condition theiry strategy on anything else than the state at date t.

Here, we investigate the restriction imposed by MPE by allowing for any type of strategy, which gives us SPE (Subgame Perfect Equilibrium). But repeated games usually have many SPE so we focus here on the SPE that is the best for the elite. This SPE will prevent a revolution for the largest possible set of parameter values.

In order to do so, we take a simplified version of the model, where there is no possibility of Democratization and the question is whether the elite can commit to a tax rate such that the poor do not want to initiate a revolution. In the MPE of this simplified version, you find that there is a threshold μ^{**} of μ^h such that:

$$\mu^h < \theta \text{ and } \mu^h < \mu^{**} \Rightarrow \text{a revolution occurs at } \mu = \mu^h \text{ at whatever tax rate the elite set.}$$

We focus here on this region, and set $\mu^h < \theta$ and $\mu^h < \mu^{**}$. ***In MPE, the elite cannot prevent a revolution. Can they in a SPE?***

Let $\mathcal{H}^{t-1}$ the set of all possible histories of play up to $t - 1$.

Strategy for the elite

$$\sigma^r : \begin{array}{lcl} \{\mu^h, \mu^l\} \times \mathcal{H}^{t-1} & \to & [0,1] \\ (\mu, h^{t-1}) & \mapsto & \tau \end{array}$$

Strategy for the poor

$$\sigma^p : \begin{array}{lcl} \{\mu^h, \mu^l\} \times \mathcal{H}^{t-1} \times [0,1] & \to & \{0,1\} \\ (\mu, h^{t-1}, \tau) & \mapsto & \rho \end{array}$$

SPE is the following:

$\{\sigma^r, \sigma^p\}$ such that σ^p and σ^r are best-responses to each other for all possible histories $h^{t-1} \in \mathcal{H}^{t-1}$ and prior actions taken within the same stage game.

Candidate Equilibrium We want to check the existence of an equilibrium of this type:

- The elite commit to a tax schedule $[\tau^l, \tau^h]$ (tax rate set in state μ^l, μ^h respectively) after any history on the equilibrium path. Otherwise they set $\tau = 0$
- The poor commit not to initiate a revolution ($\rho = 0$) after any history and state game current play on the equilibrium path. Otherwise, the poor initiate a revolution the first time the state is $\mu = \mu^h$ after they observe an off-path event.

Question 9 Let $V^i(\mu, [\tau^l, \tau^h])$ be the value for type i, in state μ, if the elite choose the tax schedule $[\tau^l, \tau^h]$, and the poor do not initiate a revolution. Prove that:

$$V^r(\mu^l, [\tau^l, \tau^h]) = \frac{y^r + (1-\beta q)(\tau^l(\bar{y} - y^r) - C(\tau^l)\bar{y}) + \beta q(\tau^h(\bar{y} - y^r) - C(\tau^h)\bar{y})}{1-\beta} \tag{1}$$

Incentive Compatibility for the elite The question is whether a schedule $[\tau^l, \tau^h]$ can be credibly promised by the elite, taking as given the candidate strategy of the poor.

Question 10 Now, assume the elite have deviated. What tax rate do they set? Let $V_d^r(\mu^l)$ the value for the rich once they have deviated. Prove that:

$$V_d^r(\mu^l) = \frac{y^r}{1-\beta(1-q)} \tag{2}$$

A necessary condition for τ^l to be incentive compatible is thus:

$$V^r(\mu^l, [\tau^l, \tau^h]) \geq V_d^r(\mu^l) \tag{IC^r}$$

Besides, the tax rate $\tau^h \leq \tau_p^*$ in the state $\mu = \mu^h$ is automatically credible (think about why).

Incentive Compatibility for the poor

Off-Path It is always incentive-compatible for the poor to initiate a revolution in the state $\mu = \mu^h$ (remember $\mu^h < \min(\theta, \mu^*)$), and not to initiate in $\mu = \mu^l$.

On-Path Let $V^p(\mu^h, [\tau^h, \tau^l])$ the value for the poor in state μ^h to play $\rho = 0$, taking as given the candidate strategy of the rich. We admit that:

$$V^p(\mu^h, [\tau^h, \tau^l]) = \frac{y^p + \beta(1-q)(\tau^l(\bar{y} - y^p) - C(\tau^l)\bar{y}) + (1-\beta(1-q))(\tau^h(\bar{y} - y^p) - C(\tau^h)\bar{y})}{1-\beta} \tag{3}$$

The Incentive Constraint for the poor is thus:

$$V^p(E, \mu^h, [\tau^l, \tau^h]) \geq V^p(R, \mu^h) \tag{IC^p}$$

with $V^p(R, \mu^h)$ coming from Question 2.

Maximization Problem for the elite The SPE that is the best for the elites, starting in the state μ^l, can be characterized as the solution of the following maximization problem (P) :

$$\max_{(\tau^l,\tau^h)} \quad V^r(\mu^l,[\tau^l,\tau^h]) \quad s.t. \quad \begin{array}{l}(IC^r)\\(IC^p)\end{array}$$

We admit that the cheapest way for the elite of providing utility of $V^p(R,\mu^h)$ to the poor is to set a constant tax rate, (remember tax-smoothing arguments, impossible in MPE as the elite cannot credibly commit to redistribute in state μ^l). So we focus on $\tau^l = \tau^h \equiv \tau^{SPE}$.

Question 11 Show that it is optimal for the elite to set τ^{SPE} such that (IC^p) holds with equality. Deduce an equation characterizing τ^{SPE} in terms of primitives of the problem.

Question 12 What is the maximal tax rate $\bar{\tau}$ that the elite can credibly promise? (Use (IC^r)).

Question 13 Is there a solution to the problem of the elite? Answer as a function of μ^h.