Liquidity Constrained Markets versus Debt Constrained Markets

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more idiosyncratic risk than complete, frictionless Arrow-Debreu markets

Hayashi [1985], Zeldes [1989], Backus, Kehoe, and Kydland [1992]

- liquidity constraints: consumers save a single asset that they cannot sell short

  Bewley [1980], Dumas [1980], Townsend [1980], Scheinkman and Weiss [1986], Abel [1990], Kehoe, Levine, and Woodford [1992]

- debt constraints: consumers cannot borrow so much that they would want to default

  Schechtman and Escurdero [1977], Manuelli [1986], Kehoe and Levine [1993], Alvarez and Jermann [1997], Kehoe and Perri [1998] and Krueger and Perri [1998]

in both models

- incomplete insurance against idiosyncratic shocks
- interest rates lower than subjective discount rates

stochastic environment models have quite different persistence

- debt constrained model less persistence than liquidity constrained model

The Environment

$t = 0, 1, \ldots$.

two types of consumers $i = 1, 2$ continuum of each type

single consumption good *x*

representative consumer of type *i* consumes x_t^i in period *t*

infinite vector of consumption is $(x_0^i, x_1^i \ldots) \in \ell_\infty^{++}$

utility $U(x_0^i, x_1^i \ldots) = (1-\delta)\sum_{t=0}^{\infty} \delta^t u(x_t^i)$

period utility function twice continuously differentiable

$Du(x) > 0$

$Du(x) \to \infty$ as $x \to 0$

$D^2 u(x) < 0$

δ satisfies $0 < \delta < 1$

two types of capital

- human capital (or labor)
- physical capital (trees or land)

$w_t^i \in \{\omega^b, \omega^g\}$ flow of services from endowment of one unit of human capital held by type *i* consumer in period *t*

$\omega^b < \omega^g$

perfect negative correlation

if one type of consumer has good productivity, the other has bad

so there are two aggregate states

two models:

- simple model – deterministic alternation
- stochastic model – Markov random switching

one unit of physical capital in the economy
physical capital durable and returns $r > 0$ units of consumption per period
trees, *r* amount of consumption good produced every period by a tree

[$r = 0$, physical capital is fiat money, but we do not allow this case]

type *i* holds share θ_t^i of the capital stock at the beginning of time *t*
initial physical capital holdings θ_0^i

social feasibility

supply of consumption good is sum of the individuals' productivity, plus return on the single unit of physical capital

$$\omega = \omega^g + \omega^b + r$$

$$x_t^1 + x_t^2 \le \omega^g + \omega^b + r = \omega$$

$$\theta_t^1 + \theta_t^2 \le 1.$$

Market Arrangements

liquidity constrained economy

intertemporal trade only by exchanging real capital

consumption good is numeraire
price of physical capital in period *t* is v_t

consumer problem

$$\max\,(1-\delta)\sum_{t=0}^{\infty}\delta^t u(x_t^i)$$

subject to

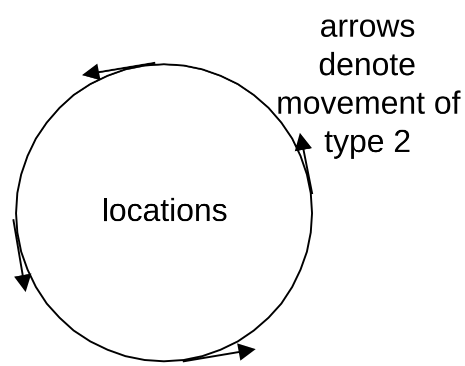
$$x_t^i + v_t\theta_{t+1}^i \le w_t^i + (v_t + r)\theta_t^i$$

$$\theta_t^i \ge 0,\quad \theta_0^i \text{ given}$$

$$t = 0, 1, \ldots$$

- physical capital can be held only in nonnegative amounts
- no securities or other assets that can be traded

physical interpretation
trade takes place on circle
only consumers at same location can trade
type 1 consumers do not move
type 2 consumers move counter-clockwise and never return to the same location



debt constrained economy

sale and purchase of insurance contracts

consumers have the option of going bankrupt and reneging on all existing debts
can be excluded from all further participation in intertemporal trade and physical capital can be seized

human capital is assumed to be inalienable - cannot be taken away

requires a credit agency, a government, or some central authority to track who has gone bankrupt seize capital and enforce exclusion from borrowing and lending individual rationality constraint

$$(1-\delta)\sum_{\tau=t}^{\infty}\delta^{\tau-t}u(x_{\tau}^{i}) \geq (1-\delta)\sum_{\tau=t}^{\infty}\delta^{\tau-t}u(w_{\tau}^{i})$$

value of continuing to participate no less than value of dropping out

absence of private information implies no consumer actually goes bankrupt in equilibrium

no one will lend so much to consumers that they will choose bankruptcy

very unlike the incomplete markets bankruptcy models of Dubey, Geanakoplos, and Shubik [1988] and Zame [1993]

complete markets
buy and sell consumption in period *t* for p_t

consumer problem

$$\max\,(1-\delta)\sum_{t=0}^{\infty}\delta^t u(x_t^i)$$

subject to

$$\sum_{t=0}^{\infty} p_t x_t^i \le \sum_{t=0}^{\infty} p_t (w_t^i + \theta_0^i r)$$

$$(1-\delta)\sum_{\tau=t}^{\infty}\delta^{\tau-t}u(x_\tau^i) \ge (1-\delta)\sum_{\tau=t}^{\infty}\delta^{\tau-t}u(w_\tau^i), \quad t = 0, 1, \ldots$$

sequential version of complete markets budget constraint

$$x_t^i + v_t \theta_{t+1}^i \le w_t^i + (v_t + r)\theta_t^i$$

$$\theta_t^i \ge -\Theta, \quad \theta_0^i \text{ given}$$

$$t = 0, 1, \ldots$$

$\theta_t^i \ge -\Theta$ rules out Ponzi schemes
unlike the liquidity constrained case where $\Theta = 0$, Θ is a positive constant chosen large enough not to constrain to borrowing

symmetric steady states

$$x_t^i = \begin{cases} x^g & \text{if } w_t^i = \omega^g \\ x^b & \text{if } w_t^i = \omega^b. \end{cases}$$

Note that by social feasibility $x^b = \omega - x^g$

Comparison of Liquidity and Debt Constrained Markets

$$f^L(x^g) = Du(x^g)(x^g - \omega^g) + \delta Du(\omega - x^g)(\omega - x^g - \omega^b).$$

Proposition 1: *A symmetric steady state x^g of the liquidity constrained economy is characterized by*

$f^L(\omega/2) \geq 0$ *and* $x^g = \omega/2$ *or*
$\omega^g > \omega/2$*,* $f^L(x^g) = 0$ *and* $x^g \in [\omega/2, \omega^g]$*.*

$$f^D(x^g) = u(x^g) - u(\omega^g) + \delta\big(u(\omega - x^g) - u(\omega^b)\big),$$

Proposition 2: *A symmetric steady state x^g of the debt constrained economy is characterized by*

$f^D(\omega/2) \geq 0$ *and* $x^g = \omega/2$ *or*
$\omega^g > \omega/2$*,* $f^D(x^g) = 0$ *and* $x^g \in [\omega/2, \omega^g]$*.*

Observations:

$$f^{D}(x^{g}) > f^{L}(x^{g})$$
$$f^{L}(\omega^{g}) > 0.$$

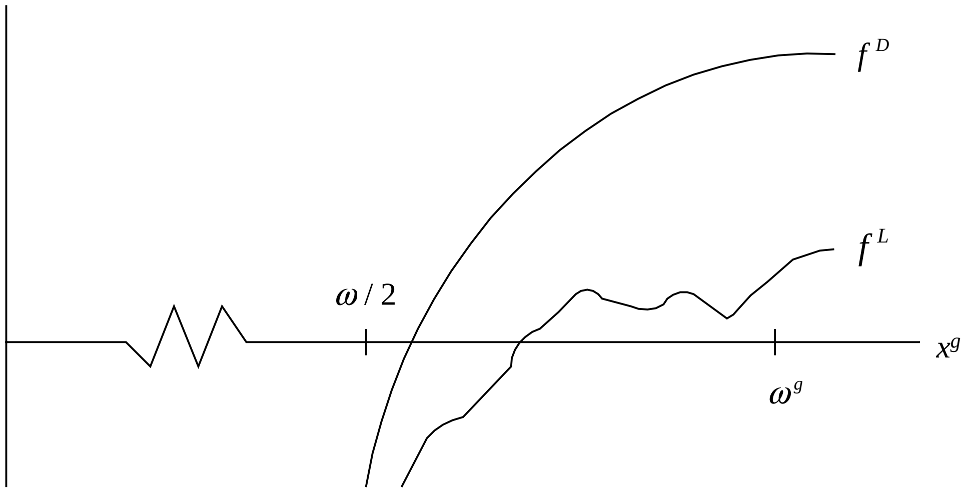


Figure 1

Proposition 3: *A symmetric steady state exists both in the liquidity constrained and in the debt constrained economy. In each case there is only one symmetric steady state.*

Interest rates

$$i = \frac{Du(x^g)}{\delta Du(x^b)} - 1.$$

At symmetric first best usual complete market answer:
interest rate equals subjective discount rate $1/\delta - 1$

otherwise $x^g > x^b$ implying interest rate lower than subjective discount rate

intuition simple: borrowers are constrained, lenders are not
desire of lenders to lend must be reduced, so market must have low rate of interest

A Stochastic Environment

$0 < \pi < 1$, which is the probability of a *reversal,* that is, a transition from the state where type 1 has good productivity to the state where type 2 has good productivity, or vice versa

$\pi = 1$ is previous deterministic alternation case

$$f^D(x^g) = (1-\delta(1-\pi))\left(u(x^g) - u(\omega^g)\right) + \delta\pi\left(u(\omega - x^g) - u(\omega^b)\right)$$

Proposition 4: *A symmetric stochastic steady state* x^g *of the debt constrained economy is characterized by*

$f^D(\omega/2) \geq 0$ and $x^g = \omega/2$ or
$\omega^g > \omega/2$, $f^D(x^g) = 0$ and $x^g \in [\omega/2, \omega^g]$.

Proposition 5: *A symmetric stochastic steady state exists in the debt constrained economy. There is only one symmetric stochastic steady state.*

Proposition 6: *If* $0 < \pi < 1$ *there is no symmetric stochastic steady state with liquidity constraints.*

Debt constrained equilibrium is simpler and exhibits less persistence.

Intuition

consumer has bad productivity for the first time
should sell some physical capital to smooth consumption
could happen repeatedly with positive probability, leading him to run out of physical capital