

Survival of the Weakest: Why the West Rules[☆]

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Abstract

We study a model of institutions that evolve through conflict. We find that one of three configurations can emerge: an extractive hegemony, a balance of power between extractive societies or a balance of power between inclusive societies - the latter being most conducive to innovation. As extractive societies are assumed to have an advantage in head to head confrontations we refer to this latter possibility as the survival of the weakest. Our contention is that the reason that the West “rules” can be traced back to two events both taking place in China: the invention of the cannon, which made possible the survival of the weakest in Europe; and the arrival of Genghis Khan, which led to the survival of the strongest in China.

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1. Introduction

There are as many theories of “why the West rules,” which is to say, why the industrial revolution took place in the West rather than, say, China, as there are historians and economists.³ One element on which there seems to be agreement (see, for example, Landes (2003), Lin (1995) and Liu and Liu (2007)) is that competition between relatively inclusive institutions such as those in Western Europe are more likely to generate innovation - and an industrial revolution - than the relatively extractive hegemonies found in China.⁴ Accepting this basic conclusion we ask: why was there competition between relatively inclusive institutions in Europe while in China we find an extractive hegemony? Why did India - made up of competing societies not an extractive hegemony - generate relatively little innovation? These are the main questions addressed in the paper.

Across history we observe a wide array of different institutions both across space and time. There are competing societies in some times and places and hegemonies in others, with political systems ranging from quite democratic to relatively autocratic - inclusive or extractive in the terminology of Acemoglu and Robinson (2012). Some, such as Diamond (1998), have argued that the geography of Europe is more favorable to competing societies than China. This not only leaves unanswered the question of India, but a careful examination of a map by Hoffman (2013) shows that the premise cannot be taken for granted.⁵ We think instead that answers to questions about hegemony and competition among institutions must be found in models of institutional evolution. Our starting observation is that historically people and institutions have more often spread through invasion and conflict than through peaceful change.⁶ In addition, significant institutional change has most often arisen in the aftermath of the disruption caused by warfare and other conflicts between societies. Consequently we are led to a theory of institutional change that arises from conflict, as in Rosenthal and Wong (2011), and not, as in Bisin and Verdier (2001) or Greif and Tabellini (2010), from internal evolution within a single society.

This paper presents a simple model of institutions that evolve through conflict. We find that one of three configurations can emerge in the long run: an extractive hegemony, a balance of power between extractive societies, or a balance of power between inclusive societies. As extractive societies are assumed to have an advantage in head to head confrontations⁷ we refer to the latter possibility as the survival of the weakest. In short, our contention is that the reason the West “rules” is that the invention of the cannon made possible the survival of the weakest in Europe -

³Many readers will note a reference to the recent book Morris (2010) in our title. McNeil (1963) and Cipolla (1965) are some earlier contributions to what is now an extensive literature.

⁴Why the character of the industrial revolution was different than earlier episodes of innovation we do not address - for discussion of this issue see Mokyr (2002) and Dutta et al (2018).

⁵Hoffman (2013) argues that a key difference between Europe and China was the role of the Catholic Church in Europe - it plays a role in his theory similar to the outsiders in ours.

⁶This was apparently true even in the earliest of times. Bowles and Choi (2013) argue that farming was initially an inferior technology to foraging and became widespread not because it was eagerly adopted by imitators, but rather because farmers had “formidable military technology” that enabled them to successfully encroach on foragers.

⁷We will explain why this assumption is consistent with the literature that argues that democracies are more effective than autocracies at conflict.

leading to competing inclusive societies - while the arrival of Genghis Khan led to the survival of the strongest in China - leading to extractive hegemony.

Since extractive institutions generally levy higher taxes and have larger armies, if evolution is driven by conflict why do these “strong” extractive institutions not predominate over “weaker” inclusive institutions? In our earlier work Levine and Modica (2013) we suggested that the answer was to be found in the presence of outsiders: while models of evolutionary conflict generate a tendency towards hegemonies, interfering outsiders such as English or Central Asians made it difficult to form hegemonies in Europe and India. Unfortunately that theory left unanswered the question of what happens when hegemonies do not form and why the record of innovation in India is so much different than that in Europe.

Here we re-examine the evolutionary conflict model. We simplify the model by omitting the details of conflict found in Levine and Modica (2013) and strengthen it along the lines suggested in Levine and Modica (2017) by adding a “home field advantage” for a society defending its own land. In this model two societies with two possible institutions - inclusive or extractive - compete through conflict. Each society has two groups - commercial elites and military elites⁸ - and conflicts occur based on the incentives of these groups. In a head to head contest between an extractive and inclusive society the extractive society is assumed to be more likely to prevail. Never-the-less we show that competition between inclusive societies may persist in the long-run. The circumstances that favor this are strong outsiders - as in Levine and Modica (2013) - and a military technology in which strong defensive forces are important. After we develop the basic model and prove our main theorem characterizing stochastically stable states (in the sense of Young (1993)) we apply it to see how well our two variables - outsiders and military technology - explain the history of hegemony and institutions (and by implication innovation). We claim that an important difference between early medieval military technology and later military technology is that the advent of the cannon made it impossible to defend with little effort behind a secure wall and made defense a far more demanding endeavor. We think that this, together with the presence of strong outsiders, made possible the survival of the weakest in Europe. On the other hand, we will argue, the arrival of Genghis Khan and the building of the Mongolian Empire which weakened outsiders in China was decisive for the development of the extractive hegemonies which prevailed there.

2. The Model

Two societies (or countries) with the same technology contend over land and the people, physical capital, and other resources that reside there. There are two units of land, one for each society. There are two possible configurations: a *balance* of power in which each society occupies its own unit of land and a *hegemony* in which one society, the *occupier*, occupies both units of land and the other society is referred to as the *occupied*.

⁸During pre-industrial revolution period we are interested in the masses - the peasant farmers - never had political power for any substantial length of time in any society.

There are two groups in each society: the *commercial elites* and the *military elites*. There are two types of institutions, inclusive institutions w and extractive institutions s . Roughly speaking with inclusive institutions the commercial elites have the upper hand, while with extractive institutions the military elites have the upper hand. Depending on circumstances either society may have either type of institution. There are five possible states of the system: $z \in Z = \{w, s, ww, sw, ss\}$. The first two correspond to a hegemony in which the occupier has inclusive and extractive institutions respectively and the remaining three correspond to a balance of power in which both have inclusive, one has extractive, the other inclusive, and in which both have extractive institutions.

Conflict between societies takes place over time $t = 1, 2, \dots$. At the beginning of period t there is a status quo given by the state from the previous period z_{t-1} . A game between the two groups in the two societies is played and the outcome determines the state z_t in the current period. The particular game depends upon the status quo z_{t-1} and a iid random shock. It takes place in two stages. In the first stage only one of the four groups is *active* and may decide to initiate a conflict to achieve a particular *goal*. The decision is based on a stochastic utility shock. If the active group is part of an occupied society the conflict is a *rebellion* to liberate their land and the goal is to install particular institutions there; thus if the rebellion is successful the hegemonic state will transit to a balance of power. If the active group is part of a balance of power the conflict is to *attack* the other society and the goal is to occupy their land; in this case success will result in a hegemony. If the active group chooses not to initiate a conflict the status quo remains unchanged and $z_t = z_{t-1}$. If the active group initiates a conflict a second stage simultaneous move game is played. The active group initiating the conflict is designated as the *aggressor* and one group from the opposing society is the *defender*. Each simultaneously decides the level of effort to devote to the conflict and these effort levels stochastically determine the new state. All of the groups are myopic in the sense that they care only about the consequences of their actions in the current period. All random events are iid conditional on the state: they do not otherwise depend on history.

Outsiders

In addition to the four decision making groups of *insiders* there are a number of *outsiders* whose strength relative to the insiders is denoted by $\eta > 0$. These outsiders do not make decisions but help determine in an important way the environment in which conflict takes place. They represent the global picture of the context in which the two societies contend: they represent societies and people outside of the model who are protected from the insiders. They may be protected by asymmetrical geographical barriers or by superior force. They represent forces that may be able to “get at the insiders” but cannot easily be “got at by the insiders.” Both geography and technology matter: the English channel was not a barrier to continental invasion given English and Roman technology in Julius Caesar’s time. After 1400 naval technology and standing navies favored strongly the short coastline of England over the long coastline of continental Europe so that England could interfere easily in the continent but could not so easily be invaded from the continent. Hence in the centuries long conflict between France and Austria/Germany the country of England was effectively an outsider.

Our basic hypothesis is that outsiders are disruptive of hegemony but supportive of a balance of power. As discussed in detail in Levine and Modica (2017) this is in broad accordance with historical facts. The most notable example is the role of England in maintaining a balance of power on the continent, which is well documented and is notorious for its complete cynicism.

Outsiders can be disruptive to hegemony in a variety of ways. First, they may provide a refuge in which governments-in-exile can regroup and in which rebel forces indeed may accumulate. For example, in 1994 the Hutu hegemony in Rwanda along with the genocide it incited came to an end when rebel Tutsi forces invaded from Uganda where they had safe haven. Second, outsiders may provide direct support in the form of weapons or even troops. For example, in the final battle of the US revolutionary war in which the British were decisively defeated the rebel force consisted of 11,000 US soldiers - together with 8,000 French soldiers and crucially the support of a French naval fleet.

We now describe in greater detail the game and payoffs: the model is simple and stylized. Subsequently we analyze the robustness of the main results to departures from this basic model.

The Initiation of Conflict

In a balance of power each society has an equal chance of being active. In a society with inclusive institutions the active group is the commercial elite. In a society with extractive institutions the active group is the military elite. The goal is to occupy the land belonging to the other society and install the active group's institutions there.

In an inclusive hegemony the active group is the occupied commercial elite. In an extractive hegemony the active group is the occupied military elite. There are two possible goals. With probability $1 > G(z_{t-1}) > 0$ the goal is to revolt and install inclusive institutions and with the remaining probability the goal is to revolt and install extractive institutions. The probability may depend upon current institutions: it may be that when the commercial elites are calling the shots it is more likely that the goal will be revolt to inclusive institutions than when the military elites are calling the shots. Notice that we assume an element of culture inherited from the occupying power: groups in the occupied territory are assumed to operate under the institutions imposed on them by the occupier. In a society run by generals it is generals who are most likely to rebel. In a society where the support of the commercial elites is essential a revolt is only possible with their support. We refer here, in particular, to the fact that in the process of decolonization the support of the commercial elites played an important role despite the fact that in precolonial times they had little voice. Notice, however, that a revolt may adopt either type of institution depending upon the goal. It is possible, for example, that the commercial elites would agree to extractive domestic institutions in return for liberation from foreign domination by inclusive institutions.

Once the active group and goal are determined an iid random utility shock \tilde{u} occurs: this is standard for a random discrete choice model. The active group then decides whether or not to initiate conflict - to attack or revolt. If the active group decides not to initiate a conflict the game ends and the state remains unchanged. In this case $z_t = z_{t-1}$ and the utility of all groups is that in the status quo. If the active group decides to initiate conflict the utility of the active group is

increased by \tilde{u} , the current state z_t is randomly determined through conflict resolution, and the utility of all groups is determined by the current state (as specified shortly) minus the costs of conflict plus the utility shock for the active group.

As conflict - at least in the sense of an all-out revolt or attempt to occupy a foreign nation - is rare, we assume that the utility shock is with high probability negative. If \tilde{u} is very negative the active group will not choose to initiate a conflict, so it is only the upper tail of this random variable that matters. We assume this has an exponential form given by three parameters $U > 1$, $0 < P < 1$ and $\sigma > 0$ so that if $v \geq -U$ then $\Pr(\tilde{u} \geq v) = Pe^{-\sigma(v+U)}$. With probability $1 - P$ the shock is smaller than $-U$ and no conflict is initiated. The parameter σ is a scale parameter for the utility shock distribution. If it is large the probability of a shock much bigger than $-U$ is very small. We will be interested in the case in which σ is large.

Conflict Resolution

Next we describe the simultaneous move game following the decision of the active group to initiate a conflict. If conflict takes place the active group - now called the *aggressor* - determines the level of effort $1 \geq x_a \geq 0$ to devote to the conflict. In a balance of power the defender is the commercial elite if the society under attack has inclusive institutions and the military elite if the society under attack has extractive institutions. In an inclusive hegemony the defender is the occupier commercial elite; in an extractive hegemony the defender is the occupier military elite. The defender determines a level of effort $1 \geq x_d \geq 0$ to devote to the conflict. Each contestant group $i \in a, d$ faces a quadratic cost of effort provision $C(x_i) = (\gamma/2)x_i^2$ where $\gamma \geq 1$. Note that the two groups who are neither aggressor nor defender do not bear any cost of conflict - although if they did it would not matter since they have no decision making power. Note also our base assumption that every society regardless of its type faces the same cost of raising resources. Our explanation of social outcomes - in contrast to Hoffman (2013)'s theory of the great divergence after 1600 - does not rest on the idea that there are systematic differences in the cost of raising resources due to social organization.⁹

Let $\zeta \in \{h, b\}$ be an indicator of whether the state is hegemonic or a balance of power. The probability the aggression succeeds depends on the resources committed by the contestant groups and is given by a conflict resolution function¹⁰

$$\pi(x_a, x_d) = \Pi_\zeta(\eta) + \alpha(x_a - [(1 - \varphi)x_d + \varphi])$$

where $0 \leq \varphi \leq 1$, $\Pi_\zeta(\eta) > 0$ is continuous with $\Pi_h(\eta)$ strictly increasing in η , $\Pi_b(\eta)$ strictly decreasing in η and $\alpha > 0$. We assume, moreover, that $\max\{\Pi_h(\infty), \Pi_b(0)\} + \alpha < 1$ and $\min\{\Pi_h(0), \Pi_b(\infty)\} - \alpha > 0$ so that regardless of the choices of effort and value of φ the probability of success is positive but not certain. Finally, we assume the boundary condition that

⁹The assumption can be relaxed, we provide details in the Web Appendix.

¹⁰See Hirshleifer (2001) and Hausken (2005) for a broader discussion of the use of conflict resolution functions.

$\Pi_h(\infty) > \Pi_b(\infty)$ and $\Pi_b(0) > \Pi_h(0) + \alpha$.

Notice that if aggressor and defender employ equal effort, $x_a = x_d = x$, then $\pi(x, x) = \Pi_\zeta(\eta) - \alpha\varphi(1 - x)$ which increases from $\pi(0, 0) = \Pi_\zeta(\eta) - \alpha\varphi$ to $\pi(1, 1) = \Pi_\zeta(\eta)$. If $\Pi_\zeta(\eta)$ is small this fact represents an intrinsic advantage of being a defender. In a hegemony the defending hegemon has the advantage of greater resources (since it controls both societies) and the disadvantage of defending on foreign soil. In a balance of power the defender does not have the resources of a hegemon but does have the advantage of defending home turf. How great these advantages or disadvantages are depends on the strength of outsiders: our assumption dependence of $\Pi_\zeta(\eta)$ on η is consistent with our view of that outsiders help rebels chances of success - $\Pi_h(\eta)$ increasing - but hurt those of an aggressor in a balance of power - $\Pi_b(\eta)$ decreasing. Our boundary assumption on Π_ζ says that outsiders are potentially important in the sense that (given efforts) if they are strong enough then rebels have a better chance of success than balance of power aggressors and if they are weak then balance of power aggressors have a better chance of success than rebels. Notice the implication that there is a unique value η^* such that $\Pi_h(\eta^*) = \Pi_b(\eta^*)$.

The parameter α measures the sensitivity of the outcome to the differential effort of the two combatants. We have assumed that this is not too large. The parameter φ measures the sensitivity of the outcome to defensive effort. The coefficient on x_d is $(1 - \varphi)x_d + \varphi$ a weighted average of the defensive effort and 1. Our interpretation is that φ measures the value of fixed fortifications. The reason for this is that the benefit of fortifications is that they enable a small army to hold off a much larger force. In Masada in 66 CE, for example, a group of roughly 1,000 men women and children held off the Roman Empire for about seven years before being overcome by a military force of around 15,000. On the other hand, effective fortifications reduce the benefit of a larger defending force: it is unlikely that a Jewish force of 2,000 or 3,000 would have had much more success against the Romans than 1,000. Here the effectiveness of defense is measured by $(1 - \varphi)x_d + \varphi$ where φ captures the basic idea that with effective fortifications the defense is strong but not particularly sensitive to defensive strength. Hence our interpretation of φ as the effectiveness of fortifications.

If the aggression fails the status quo remains unchanged, $z_t = z_{t-1}$ and the utility of all groups is that in the status quo less the effort and plus the utility shock. If the status quo is a balance of power and the aggression succeeds the new state is a hegemony with the institutions of the aggressor. If the status quo is a hegemony and the aggression succeeds the new state is a balance of power in which the defender institutions are unchanged and the aggressor institutions are those determined by its goal. In all success cases the utility of all groups is that of the new state less the effort and plus the utility shock.

Incentives: Transfers and Utility

In addition to the random utility shock and conflict costs the utility of groups is determined by the current state z_t . From the economic point of view the two groups in each society are characterized by a *transfer* of resources from the commercial elites to the military elites. Extractive societies are defined so that this transfer is larger than in inclusive ones. So there are two possible

transfer levels representing a transfer from the commercial elites to the military elites on each unit of land; we normalize high transfers to 1, and low transfers are $0 < \tau < 1$. In a balance of power the military elites receive the transfers from their own land, so for example in a w -type society we can write the transfer vector as $(-\tau, \tau)$. In a hegemony the occupier military elites receive the transfers from both units of land (so the occupied military elites receive nothing); high transfers are always taken from the occupied commercial elites; and the occupier commercial elites pay τ in an inclusive hegemony and 1 if the hegemony is extractive; so for example in an extractive hegemony the transfer vector is $(-1, 0)$ in the occupied society and $(-1, 2)$ for the occupiers (the occupier military elites collect all the transfers). The transfers are summarized in the following table:

	s	w
balance	$-1, 1$	$-\tau, \tau$
occupier	$-1, 2$	$-\tau, 1 + \tau$
occupied	$-1, 0$	$-1, 0$

While it is convenient to describe the benefits and costs of occupation in terms of transfers, the key features are that there is a preference against foreign rule by both commercial elites and military elites and that occupier military elites benefit from occupation at the expense of the occupied military elites. For example, it may be that commercial elites dislike foreign rule because of issues of language or distaste for foreigners. It may be that occupier military elites enjoy the status that comes with ruling over a larger territory. We take as a relatively neutral assumption that commercial elites are indifferent between being exploited by foreigners and by domestic military elites and that occupier military elites receive the same benefits from occupied territories as they do from extractive institutions in their domestic territory.

Equilibrium

An equilibrium is the stochastic process in which a Nash equilibrium occurs within each period. We will show that this equilibrium is unique and depends only on the state z in the previous period. Likewise the probability of the current state conditional on the within period equilibrium depends only on the previous state. Hence an equilibrium is a Markov process on the state space $Z = \{w, s, ww, sw, ss\}$. From what we have seen the possible transitions other than remaining at the status quo are the following:

$$w \rightarrow ww, sw \quad s \rightarrow ss, sw \quad ww \rightarrow w \quad sw \rightarrow s, w \quad ss \rightarrow s$$

Notice that there is a positive probability of remaining in place and a positive probability of each of the eight feasible transitions. Hence the process is aperiodic and ergodic. This means that every state is visited infinitely often and there is a well-defined long-run frequency with which that state occurs. We denote this unique ergodic probability distribution over the state space by μ_σ . From Young (1993) we also know that as $\sigma \rightarrow \infty$ the ergodic distributions μ_σ have a unique limit μ .

Those states that have positive probability in the limit distribution μ are called *stochastically stable*. When σ is large and there is a unique stochastically stable state this means it has a frequency near one and the other states a frequency near zero. While the ergodic nature of the Markov process implies that with probability one there will be a departure from the stochastically stable state the amount of time before that departure will be relatively long and the length of the departure will be relatively short. In short, stochastically stable states are those which are observed “most of the time” when σ is large. As we are interested in the case where σ is large - that is serious conflict is infrequent - we will characterize the stochastically stable states.

3. Stochastic Stability

We now state the main result of the paper which characterizes “typical” institutional configurations. As to the “survival of the weakest”, as anticipated in the introduction it emerges as a long run possibility with strong outsiders and ineffective fortifications. The role of the φ parameter is more subtle than it may appear: it is true that as it becomes lower more defense effort is needed, but it is equally true that the probability of success with equal forces increases - recall that $\pi(x, x) = \Pi_\zeta(\eta) - \alpha\varphi(1 - x)$ - so the result is not due to the mechanics of conflict directly. Rather, technology affects the economic incentives to subvert the status quo order in the conflict subgame, and these turn out to be weaker with low φ .

Theorem 1 (Main Theorem). *For generic values of the parameters there is a unique stochastically stable state. Only s, ss, ww can be stochastically stable; w and sw cannot. There exists a $0 < \tau^* < 1$ and a strictly decreasing function $0 < \varphi_\tau < 1$ such that*

1. if $\tau > \tau^*$ or $\varphi > \varphi_\tau$ then ww is not stochastically stable, with s stochastically stable for $\eta < \eta^*$ and ss stochastically stable for $\eta > \eta^*$
2. if $\tau < \tau^*$ and $\varphi < \varphi_\tau$ then ss is not stochastically stable and there is a positive continuous strictly decreasing function $\eta(\varphi) \leq \eta^*$ with s stochastically stable for $\eta < \eta(\varphi)$ and ww stochastically stable for $\eta > \eta(\varphi)$.

We prove the result later in the section. To parse this result, consider first the case $\tau > \tau^*$, which is to say inclusive institutions do not offer such a great advantage over extractive institutions. In this case we find that inclusive institutions are never stochastically stable. Roughly: the commercial elites are unwilling to make much effort to defend inclusive institutions that are not all that inclusive. We do not find this fact terribly interesting: there are institutions of varying degrees of inclusiveness - and our focus naturally is on whether sufficiently inclusive institutions may survive. Never-the-less this result is interesting: it implies that we will not often see “somewhat inclusive” institutions, only extractive or “strongly inclusive” institutions. There are two ways in which we can make this observation precise. We could analyze a setting with three types of institutions - that is beyond the scope of this paper. We might also consider that institutions evolve within societies. We do not propose to explicitly analyze such a model - we refer the reader to the excellent survey of Bisin and Verdier (2005) for a review of models of internal institutional change. Rather we consider the

claim made by Olson (1982) and others since that inclusive institutions tend to become less so over time. In this model if we start with parameters such that a balance of power between inclusive institutions is stochastically stable (low τ) and over time τ gradually rises we will initially see a gradual deterioration of living standards for the commercial elites - followed by a rather catastrophic collapse to extractive institutions once the threshold τ^* is crossed. This is reminiscent of Hayek (1944)'s road to serfdom. We should say, however, that evidence for such a scenario is poor. On the one hand, although current politics are turning against globalization, proposals by populist parties are in fact rather modest and scarcely propose turning the clock back to 1982. On the other hand it seems that events in China must be described as the "road away from serfdom" as current Chinese institutions hardly rival in extractiveness those in the time of the cultural revolution or the great leap forward.

As we have observed there are many possible inclusive societies with different values of τ and the same set of potential institutions available in different locations and at different times. The only interesting values of τ are those for which $\tau < \tau^*$ - other not so inclusive institutions we know will not survive. For modeling purposes it is convenient to focus on a single value of τ . This should then be the same across locations and times and should be "interesting" in the sense that $\tau < \tau^*$. Hence for the remainder of the paper we are going to focus on the case $\tau < \tau^*$, that is "strongly inclusive" institutions and ask how they fare against extractive institutions.

With $\tau < \tau^*$ a careful reading of theorem 1 shows that if $\eta < \min_\varphi \eta(\varphi)$ then only extractive hegemony is stochastically stable. If there are sufficiently few outsiders then we should generally observe extractive hegemonies - and this is true regardless of military technology. By contrast with stronger outsiders, that is, larger values of η , we will see a balance of power - but military technology determines which type. When φ is large (specifically $\varphi > \varphi_\tau$) then an extractive balance of power is stochastically stable, while if φ is small then an inclusive balance of power is stochastically stable. Large φ - effective fortifications - favors extractive institutions, while small φ - good siege technology - favors inclusive technology. To anticipate: the invention of gunpowder led to a great reduction in the effectiveness of fortifications. With small φ inclusive institutions may be stochastically stable - and an inclusive balance of power indeed emerged in Europe, but not generally speaking in China or India. We will examine this history in greater detail below.

Before getting to history the remainder of this section is devoted to proving Theorem 1: in doing so we will outline the key forces that drive the result. Since outsiders increase the likelihood of hegemonies failing and the likelihood of a balance of power surviving it is not terribly surprising that when outsiders are strong only the balance of power can be stochastically stable and when outsiders are weak only hegemonies can be stochastically stable. The less obvious but crucial point is to establish which type of balance of power or hegemony is stochastically stable in each case: weak or strong? To do this we must first analyze how the conflict subgame maps the incentives of the military elites and commercial elites to the probabilities of outcomes.

The Conflict Subgame

To analyze how incentives determine success and failure in conflict we must find the equilibrium \hat{x}_a, \hat{x}_d of the subgame in which conflict takes place. We show in the Appendix that this equilibrium is unique and does not depend on the state or strength of outsiders. It does depend on y_a the transfer benefit to the prospective aggressor from the change in state and on y_d the loss to prospective defender from the change in state. These are just the changes in transfers: for instance in the transition $s \rightarrow sw$ the aggressor is the occupied military elites with the goal of establishing an inclusive society in their land so they pass from 0 to τ meaning $y_a = \tau - 0 = \tau$; the defender is the occupier military elites, whose revenues would fall from 2 to 1, whence $y_d = 1$. Note that these values lie between 0 and 1. Define the *partial utility gain* to the aggressor as the part of the equilibrium gain that is independent of the state and strength of outsiders $u(y_a, y_d, \varphi) = \alpha(\hat{x}_a - [(1 - \varphi)\hat{x}_d + \varphi])y_a - (\gamma/2)\hat{x}_a^2$. The *overall utility gain* of the aggressor can then be written as $\Pi_\zeta(\eta)y_a + u(y_a, y_d, \varphi)$. The partial utility gain is the key measure of incentives for conflict that we need to analyze stochastic stability. We report the relevant facts proven in the Appendix:

Theorem 2. *The conflict subgame has a unique Nash equilibrium independent of the state and strength of outsiders. The total utility gain to the aggressor $\Pi_\zeta(\eta)y_a + u(y_a, y_d, \varphi)$ is non-negative, less than or equal to one and strictly increasing in y_a . The partial utility gain $u(y_a, y_d, \varphi)$ is decreasing in y_d , and satisfies $u(0, y_d, \varphi) = 0$ and $u(y_a, y_d, 1) = u(y_a, 0)$. There is a $0 < \tau^* < 1$ and a strictly decreasing function $0 < \varphi_\tau < 1$ such that the function $v(\varphi) \equiv u(1, 1 - \tau, \varphi) - u(\tau, 1, \varphi) - u(1 - \tau, 0, \varphi)$ satisfies $|v(\varphi)| < \alpha$ with $v(\varphi) < 0$ for $\tau < \tau^*$ and $\varphi < \varphi_\tau$ and $v(\varphi) > 0$ otherwise.*

Resistance and Incentives, and Proof of the Main Theorem

The key technical concept used to analyze stochastic stability is the notion of *resistance*. Under our assumptions with high probability the state remains unchanged: we are interested in the probabilities of transitions that change the state. The *ex ante* probability of a successful implementation of a particular goal has the form

$$Q \cdot \Pr(\Pi_\zeta(\eta)y_a + u(y_a, y_d, \varphi) + \tilde{u} \geq 0) = Qe^{-\sigma[U - \Pi_\zeta(\eta)y_a - u(y_a, y_d, \varphi)]}$$

where Q takes account of the probability that the group is in fact active and that a particular goal is on the table. The key point is that in our model Q is independent of σ and bounded away from 0 and 1. This enables us to analyze stochastic stability using the standard notion of resistance (see, for example, Young (1993) or Kandori, Mailath and Rob (1993)). Here we take $\epsilon = e^{-\sigma}$ so that as $\sigma \rightarrow \infty$ then $\epsilon \rightarrow 0$. The resistance r is then defined as the derivative of the logarithm of the probability with respect to the logarithm of ϵ . Roughly speaking the expected length of time before the transition takes place is $e^{-r} = e^{\sigma r}$.

What then is the resistance of $r(z \rightarrow z')$ to a transition from a state z to $z' \neq z$? The states z, z' determine who is the aggressor and defender and the incentives $y_a(z \rightarrow z'), y_d(z \rightarrow z')$ of each, along with the indicator $\zeta(z)$ of whether z is a hegemony or balance of power. Hence the resistance

depends on incentives and is given by

$$r(z \rightarrow z') = U - \Pi_{\zeta(z)}(\eta)y_a(z \rightarrow z') - u(y_a(z \rightarrow z'), y_d(z \rightarrow z'), \varphi).$$

This equation highlights a key feature of the model. The only certain thing about conflict is that it will be costly: there is no guarantee of either success or failure. Hence resistance is not driven directly by the chance of winning or losing both of which have a substantial chance of occurring. Rather it is driven by the relatively rare decision to initiate conflict. One subtle consequence of this is that the strength of outsiders does not directly impact resistance. It is not true that simply because outsiders are strong a hegemony is bound to fail or that because they are weak a balance of power must collapse. Outsiders cannot in this model initiate conflict, they are relevant only if one of the decision makers decides to initiate conflict - they are relevant only if they are invited. We can see this in the term $\Pi_{\zeta(z)}(\eta)y_a(z \rightarrow z')$ in the equation for resistance: if the aggressor has little reason to initiate a conflict then the strength of the outsiders does not matter very much. In the extreme case $y_a(z \rightarrow z') = 0$ and outsiders are irrelevant. This happens, for example, in a weak society in a balance of power: the commercial elites who control the decision making have no reason to attack their neighbor and do not invite the outsiders.

We can now give the technical details of the proof of the main theorem.

Proof of Theorem 1. The resistance equation enables us to compute the resistances of all transitions. To do so we first observe that the Markov process has a birth-death like structure: each state can at most move only to one of two adjacent states. From ss the only non-trivial transition is to s . From s it is possible only to transition back to ss or forward to sw , and so forth. In the table below we lay out the resistances of the non-trivial feasible transitions. We indicate first the transition then the benefit from the transition to the commercial elites and military elites respectively. This is divided into two columns for occupier or occupied; if the initial state is a hegemony these are the hegemony's four groups; in case the initial state is a balance of power occupier versus occupied refers to the situation after transition - for example in the transition $sw \rightarrow s$ the inclusive society's groups end occupied and extractive society's groups become occupiers; the s military elites have $y_a = 1$ while the w military elites have $y_d = \tau$. The asterisks denote which group is making the decision about attack or paying for defense. Note that the defender never gains from a transition and consequently the negative values in the table always correspond to the defender. The value y_d is the negative of the defender "benefit" reported in the table: it represents the loss avoided when the defenders prevent the transition from taking place. The final column reports the resistance.

transition	occupier	occupied	$r(z \rightarrow z')$
$ss \rightarrow s$	$0, 1^*$	$0, -1^*$	$U - \Pi_b(\eta) - u(1, 1, \varphi)$
$s \rightarrow ss$	$0, -1^*$	$0, 1^*$	$U - \Pi_h(\eta) - u(1, 1, \varphi)$
$s \rightarrow sw$	$0, -1^*$	$1 - \tau, \tau^*$	$U - \Pi_h(\eta)\tau - u(\tau, 1, \varphi)$
$sw \rightarrow s$	$0, 1^*$	$-(1 - \tau)^*, -\tau$	$U - \Pi_b(\eta) - u(1, 1 - \tau, \varphi)$
$sw \rightarrow w$	$0^*, 1$	$0, -1^*$	U
$w \rightarrow sw$	$0^*, -1$	$0^*, 1$	U
$w \rightarrow ww$	$0^*, -1$	$1 - \tau^*, \tau$	$U - \Pi_h(\eta)(1 - \tau) - u(1 - \tau, 0, \varphi)$
$ww \rightarrow w$	$0^*, 1$	$-(1 - \tau)^*, -\tau$	U

The attribute of a state that determines the relative time the process spends in it is the *modified radius*. We denote by $R(z)$ the modified radius of the state z . The critical fact is that the stochastically stable states are exactly those with the greatest modified radius. The general definitions are given in Levine and Modica (2016), section 6.3, we sketch here the concept to apply it to the present setting. We say that a collection of states form a *circuit* if any two of them are connected by a least resistance path. The states can be partitioned into circuits, and then one defines circuits of circuits - 2nd-order circuits we may say - by taking as modified resistance the incremental resistance needed to move from one to the other over that needed to move within the circuit. It is possible to define even higher order circuits: the highest order circuit in this model is the 3rd order circuit consisting of all the states.

We start by determining the structure of circuits. We observe that $r(s \rightarrow sw) = U - \Pi_h(\eta)\tau - u(\tau, 1, \varphi) > U - \Pi_h(\eta) - u(1, 1, \varphi) = r(s \rightarrow ss)$ by the monotonicity property of $u(y_a, y_d, \varphi)$. This implies that ss and s form a circuit. Also by monotonicity we have $r(w \rightarrow ww) = U - \Pi_h(\eta)(1 - \tau) - u(1 - \tau, 0, \varphi) < U - u(0, 0, \varphi) = r(w \rightarrow sw)$ so that ww and w form a circuit. The only remaining question is at the next level which circuit sw joins. It will join the circuit to which it has the least resistance of reaching. By monotonicity $r(sw \rightarrow s) = U - \Pi_b(\eta) - u(1, 1 - \tau, \varphi) < U - u(0, 1, \varphi) = r(sw \rightarrow w)$ so the resistance of joining the $ss \leftrightarrow s$ circuit is less than joining the $w \leftrightarrow ww$ circuit. We may summarize the situation of circuits by the following diagram: $[(ss \leftrightarrow s) \leftrightarrow sw] \leftrightarrow (w \leftrightarrow ww)$, where round brackets mean “in a circuit with” and square brackets mean “in a 2nd order circuit with” and the entirely forming a 3rd order circuit.

In general we compute modified radii by moving between circuits adding in the incremental cost of moving up to the next level until the highest level circuit is joined.¹¹ This can be complicated since there may be many ways of moving between states, but in this model because of the birth-death like structure matters are simplified since there is exactly one way to get from one state to another. Start with s . To get to the highest level we must move to sw then to the other circuit at w . The radius of s is $r(s \rightarrow ss)$. To this we add the incremental resistance of moving to sw which is $r(s \rightarrow sw) - r(s \rightarrow ss)$ and the incremental resistance of moving to w which is

¹¹This is similar to the computation of Ellison (2000)'s co-radius.

$r(sw \rightarrow w) - r(sw \rightarrow s)$. Denoting the modified radius by $R(z)$ we then have

$$\begin{aligned} R(s) &= r(s \rightarrow ss) + r(s \rightarrow sw) - r(s \rightarrow ss) + r(sw \rightarrow w) - r(sw \rightarrow s) \\ &= r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s) \end{aligned}$$

Notice how the backward looking part $r(s \rightarrow ss)$ cancels out. This is a general property in a birth-death like model in which movement is one step in a single dimension.

We may apply this idea to all five states to compute the modified radius. The second column shows how the modified radius depends upon the resistances, the third how they depend on the model parameters. In that computation we make use of the fact that $u(0, y_d, \varphi) = 0$.

state	R	R
ss	$r(ss \rightarrow s) - r(s \rightarrow ss) + r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s)$	$U + \Pi_h(\eta)(1 - \tau) + [u(1, 1 - \tau, \varphi) - u(\tau, 1, \varphi)]$
s	$r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s)$	$U + \Pi_b(\eta) - \Pi_h(\eta)\tau + [u(1, 1 - \tau, \varphi) - u(\tau, 1, \varphi)]$
sw	$r(sw \rightarrow w)$	U
w	$r(w \rightarrow sw)$	U
ww	$r(ww \rightarrow w) - r(w \rightarrow ww) + r(w \rightarrow sw)$	$U + \Pi_h(\eta)(1 - \tau) + u(1 - \tau, 0, \varphi)$

Since $R(ww) > U$ we immediately see that neither sw nor w is stochastically stable. We compare $R(ss) - R(s) = \Pi_h(\eta) - \Pi_b(\eta)$ and $R(ss) - R(ww) = u(1, 1 - \tau, \varphi) - u(\tau, 1, \varphi) - u(1 - \tau, 0, \varphi) = v(\varphi)$. We see that between ss and s stochastic stability is determined entirely by whether $\eta > \eta^*$ or $\eta < \eta^*$. Between ss and ww we see that stochastic stability is determined entirely by $v(\varphi)$. Putting this together with Theorem 2 the only fact remaining to prove in the final case for η sufficiently small s is in fact stochastically stable, that is, $R(s) = U + \Pi_b(\eta) - \Pi_h(\eta) + \Pi_h(\eta)(1 - \tau) + [u(1, 1 - \tau, \varphi) - u(\tau, 1, \varphi)] > U + \Pi_h(\eta)(1 - \tau) + u(1 - \tau, 0, \varphi) = R(ww)$. This will hold for sufficiently small η if $\Pi_b(0) - \Pi_h(0) + v(\varphi) > 0$ which is true since $|v(\varphi)| < \alpha$ and we assumed that $\Pi_b(0) - \Pi_h(0) > \alpha$. \square

Driving Force of the Model

The key result of the model is that when outsiders are strong and fortifications weak a balance of power ww between inclusive societies is stochastically stable. Since strong outsiders clearly favor a balance of power over hegemony, the crucial thing to understand is why ww can beat ss , or from a technical point of view, why the modified radius $R(ww)$ can be larger than $R(ss)$.

To understand better these modified radii, observe that each resistance has two components, an “outsider” component of the form $U - \Pi_\zeta(\eta)T_a$ and an incentive component of the form $r = -u(T_a, T_d, \varphi)$ where $T_a, T_d \in \{0, \tau, 1 - \tau, 1\}$ are the relevant transfers. When we take the difference $R(ss) - R(ww)$ all the outsider components cancel out - not surprising since both balances of power are on an equal footing with respect to the influence of outsiders - so we can focus on the incentive component. That is, we may compare $\underline{R}(ss) = \underline{r}(ss \rightarrow s) - \underline{r}(s \rightarrow ss) + \underline{r}(s \rightarrow sw) + \underline{r}(sw \rightarrow w) - \underline{r}(sw \rightarrow s)$ and $\underline{R}(ww) = \underline{r}(ww \rightarrow w) - \underline{r}(w \rightarrow ww) + \underline{r}(w \rightarrow sw)$.

Turning first to the incentive component of the extractive balance of power $\underline{R}(ss)$ we see first that $\underline{r}(ss \rightarrow s) = \underline{r}(s \rightarrow ss) = u(1, 1, \varphi)$ is a wash. That is, the incentive for the military elite

is always the transfer 1 from the commercial elites - this represents a gain to the aggressor and a loss to the defender regardless of whether there is a hegemony or balance of power. Hence we can write $\underline{R}(ss) = \underline{r}(s \rightarrow sw) + [\underline{r}(sw \rightarrow w) - \underline{r}(sw \rightarrow s)]$. This in turn has two parts. The first part $\underline{r}(s \rightarrow sw) = -u(\tau, 1, \varphi)$ is zero if τ is zero, and since we know τ must be small this term must also be small. Hence the critical determinant of the modified radius of the extractive balance of power is $[\underline{r}(sw \rightarrow w) - \underline{r}(sw \rightarrow s)] = u(1, 1 - \tau, \varphi)$ which we refer to as the *incentive component of extractive advantage*.

To understand this better, when we account for the outsider component we can find the overall *extractive advantage* $r(sw \rightarrow w) - r(sw \rightarrow s) = \Pi_b(\eta) + [\underline{r}(sw \rightarrow w) - \underline{r}(sw \rightarrow s)] = \Pi_b(\eta) + u(1, 1 - \tau, \varphi) > 0$. This measures what happens in a head to head contest between an extractive and an inclusive society. That is, it measures which side is more likely to prevail in the state sw : if $r(sw \rightarrow s)$ is lower than $r(sw \rightarrow w)$ then the extractive society is (much) more likely to prevail over the inclusive society than the other way around. Since the extractive advantage is positive it represents the sense in which an extractive society is “strong” and an inclusive society “weak:” the former is more likely to prevail in a head to head contest.

It is the incentive component of the extractive balance of power $\underline{R}(ss)$ which must be compared to that of the inclusive balance of power $\underline{R}(ww) = \underline{r}(ww \rightarrow w) - \underline{r}(w \rightarrow ww) + \underline{r}(w \rightarrow sw) = -\underline{r}(w \rightarrow ww) = u(1 - \tau, 0, \varphi)$. That is, what provides stability to the weak balance of power is the fact that the weak hegemony collapses very fast into a weak balance of power - the occupied commercial elites have every reason to revolt and the occupying commercial elites no reason to resist. Hence the comparison between $\underline{R}(ww)$ and $\underline{R}(ss)$ boils down to comparing $u(1 - \tau, 0, \varphi)$ the incentive for occupied commercial elites to revolt against unresisting occupiers and $u(1, 1 - \tau, \varphi)$ the extractive advantage - the incentive for military elites to invade an inclusive neighbor. There are two key things about this: first, the lower is τ , that is the more inclusive is an inclusive society, the more the commercial elites want to revolt against the hegemon and the more they will resist an invasion by neighboring military elites - this of course works in favor of ww over ss and indeed for $\tau = 0$ the comparison is $u(1, 0, \varphi)$ versus $u(1, 1, \varphi)$ with the former clearly greater when $\varphi > 0$. From this as well we see clearly the role of fortifications. If φ is large then the incentives of the defender do not matter much: indeed for $\varphi = 1$ by Theorem 2 $u(1, 1 - \tau, 1) = u(1, 0, 1)$ which is larger than $u(1 - \tau, 0, 1)$, which is to say the inclusive society is less table. As φ declines and fortifications are less effective the incentive of the commercial elites to defend themselves against invading military elites increases so the extractive advantage declines relative to $u(1 - \tau, 0, \varphi)$. The point is, the incentive to defend only matters if defensive effort makes a difference to the outcome.

4. History

The theory suggests that institutions and international competition can be explained by two variables: the strength of outsiders and military technology. In particular, weak outsiders should result in extractive hegemonies regardless of military technology. Strong outsiders should result in a balance of power: between extractive states if fixed defenses are strong and between inclusive states

if fixed defenses are easily overcome. Here we see whether this idea is useful in organizing historical facts. We focus on the main centers of world population: India, China and Europe. We are constrained going back in time by lack of reliable data: for this reason we chose to take our starting point as 1 CE, including the entire history those societies that already existed at that time. It is also the case that the enormous improvements in transportation and military technology changed the world from one of militarily isolated regions into a world where Europe was able to dominate the world. This occurred around 1550 CE which we take as our ending point, again including the entire history of those societies that already existed at that time. Within this context we analyze the strength of outsiders and fortifications, the existence of hegemony, and the inclusiveness of societies.

Matching Theory with Data

To match the theory with data we need to accomplish several tasks:

1. We must locate the core areas over which there is conflict and the peripheral areas that contain outsiders. The key distinction in the model is that within the core conquest is possible, while the peripheral areas are on the one hand able to interfere in the core but relatively immune from conquest.
2. We must assess the geopolitical situation in the core. Is there a hegemony or balance of power?
3. We must assess social circumstances in the core: how inclusive are societies?
4. We must assess the strength of outsiders η in the periphery.
5. We must asses the military technology φ in the core.

In principle these are daunting tasks, but they are made much easier by the fact that - as the model requires - these variable do not change much over time, varying little from one century to the next.

Core Areas

The details of our analysis can be found in the Data Appendix. We take as core areas India, China and Europe, where the majority of the population lived. As the theory says we should observe stochastically stable states for a random length of time we use observations of variable period length based on evidence of changes. For the most part the exact start and end dates of the observations do not matter, and we adopted conventional dates used by historians. The details of the core areas in the analysis are the following:

- We take the core area of India to be the area occupied by the British during the Raj and the periphery to be Central Asia. Specifically the boundaries of the core are the Himalayas in the North, the jungles of Myanmar in the East, and Afghanistan in the West. We divide the data into five periods beginning in 200 BCE and ending in 1748 CE; the duration of these periods ranges from 2.2 to 5.2 centuries.

- We take the core area of China to be the current area of China minus the Mongolian region (the provinces of Heilongjiang, Inner Mongolia (Neimongol), and Xingjiang) and the periphery to be the Mongolian region. Specifically the boundaries of the core are the Gobi desert in the West, the Himalayan mountains in the South and the Mongolia region (including Mongolia) in the North. We divide the data into five periods beginning in 202 BCE and ending in 1839 CE; the duration of these periods ranges from 3.2 to 5.6 centuries.
- We take the core of Europe to be Western and Eastern Europe minus Britain, the Scandinavian peninsula and Russia, and the periphery to be Britain, the Scandinavian peninsula and Russia. Specifically the boundaries of the core are the sea, including the Atlantic, Mediterranean, English Channel, North Sea, Baltic Sea and Black Sea, along with the border with the former USSR, which is to say roughly the current eastern borders of Poland, Slovakia and Romania. We divide the data into five periods beginning in 149 BCE and ending in 1607 CE; the duration of these periods range from 2.4 to 4.8 centuries.

Data Overview

Each region and period is assigned an empirical state z in ss, s, ww based on whether or not there was a hegemony of the region during that period and the degree of inclusiveness of societies in that region.

The theory tells us that outsiders and fortifications matters only above and below some threshold. As the data is noisy, we categorize these each into two categories *strong* and *weak*.

The data consists of 15 observations as described in the Data Appendix. Each observations provides a binary variable for the strength of outsiders η and fortifications φ together with the empirical state $z \in \{ss, s, ww\}$. Let f denote the actual frequency of the state and \hat{f} the frequency of the stable state as determined by the exogenous variables: we report below the empirical and theoretical frequencies.

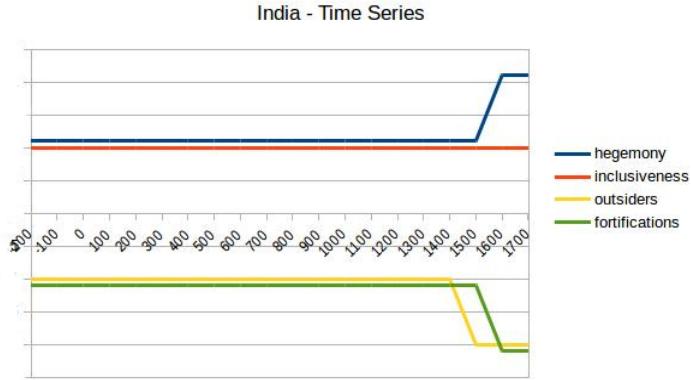
state	f	\hat{f}
ss	0.53	0.60
s	0.33	0.27
ww	0.13	0.13

Next we give the timeline of the observations together with an overview of how the theory fares.

India

In India outsiders were strong from the beginning of the era, but they are in decline by the 1500s due to the growth of core population relative to the periphery. Effective siege technology arrived late, about the same time the outsiders became weak.

During this era India was hegemonized only at the end when the Moguls arrived. There is no evidence that India was inclusive prior to independence in 1949.

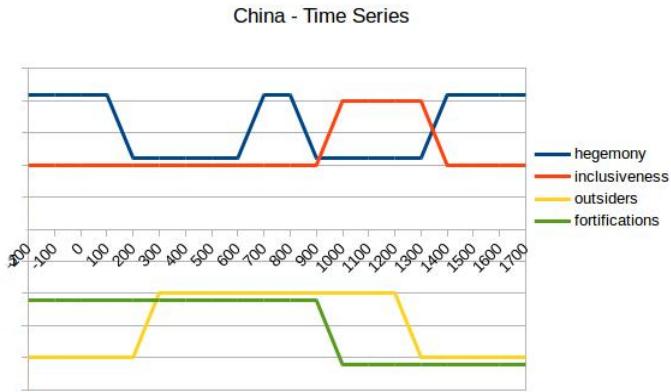


The theory predicts that India should have seen an extractive balance of power for most of the era, switching at the end to an extractive hegemony as the cannons arrived too late. This is indeed what we see in the Indian data.

China

In China outsiders were initially weak but early globalization, especially outmigration from Central Asia in the 400s increased the strength of the periphery relative to the core. Around 1300 there was an adverse demographic event leading to the relative depopulation of Mongolia so that outsiders were again weak. Effective siege technology arrived early, around 1000,

China was hegemonized during the Han dynasty until 220 CE, during the Sui/Tung dynasty between 581-907 CE, and again starting with the Yuan in 1271 CE. The only period of inclusiveness was during the Song dynasty 960-1234 CE.

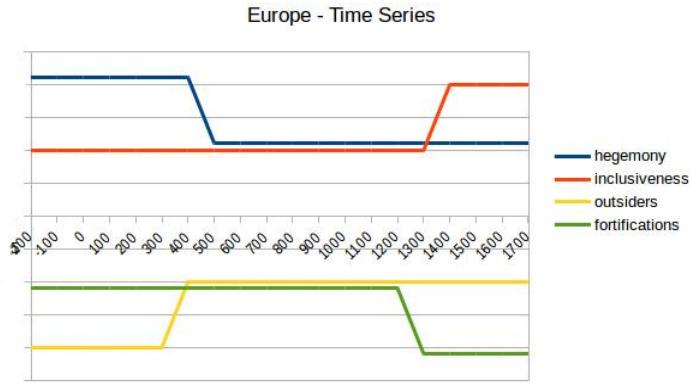


The theory predicts that we should have initially seen an extractive hegemony, followed by an extractive balance of power, followed by an inclusive balance of power due to the development of effective siege technology, with reversion to an extractive hegemony due to the weakening of the outsiders. In this case the theory fails to predict the Sui/Tung hegemony: with strong outsiders and fortifications the model predicts a balance of power.

Europe

In Europe like China outsiders were initially weak but early globalization, again, especially outmigration from Central Asia increased the strength of the periphery. Unlike China there was never a setback in the strength of the outsiders, and from around the 4th Century the influence of outsiders remained strong. Effective siege technology in the form of the cannon arrived from China in around the 14th Century.

Europe was hegemonized until the decline of the Roman Empire, and not since. Inclusiveness initially low began to rise during the Renaissance in the late 1200s.



The theory predicts that we should have seen an extractive hegemony, followed by an extractive balance of power followed by an inclusive balance of power. This is what we see in the European data.

4.1. Data Analysis

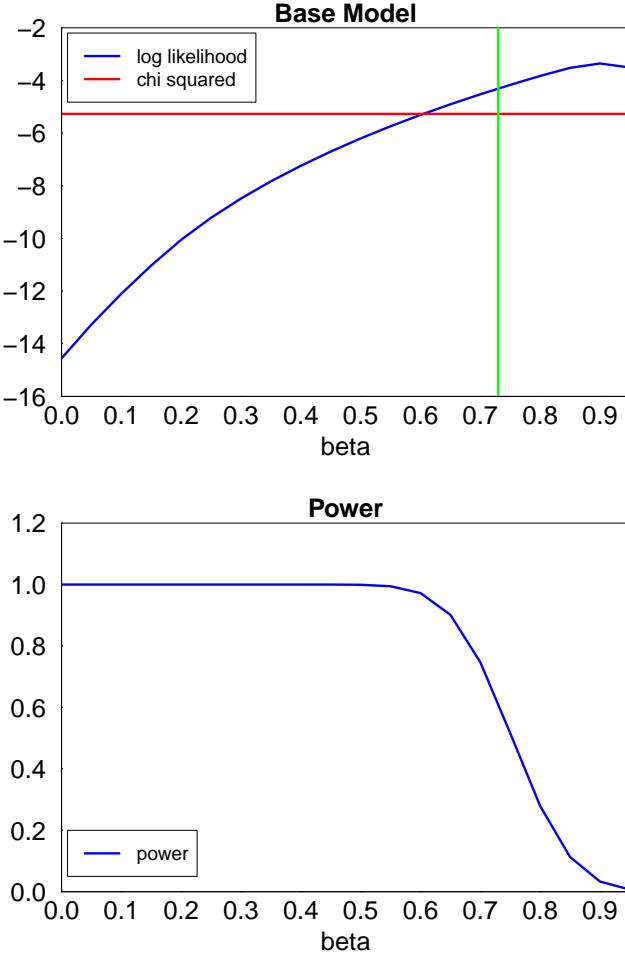
For the formal analysis recall that $z \in \{ss, s, ww\}$ is the observed state and let $\hat{z}(\eta, \varphi)$ the stochastically stable state determined by the exogenous variables. Our theoretical model says that the stochastically stable state should be observed with high probability over sufficiently long periods. We take “long periods” to mean 2.2 to 5.6 centuries. We model “high probability” empirically by assuming that with probability β the stochastically stable state $\hat{z}(\eta, \varphi)$ is observed and that with probability $1 - \beta$ the observed state is drawn randomly with probabilities $\alpha(z)/(1 - \beta)$ where $\sum_{z \in \{ss, s, ww\}} \alpha(z) = 1 - \beta$. This gives rise to the linear conditional probability model $\Pr(z|\eta, \varphi) = \alpha(z) + \beta \cdot 1(z = \hat{z}(\eta, \varphi))$.

Maximum likelihood estimation of this model gives estimates $\bar{\alpha}(ss) = \bar{\alpha}(ww) = 0$ and $\bar{\beta} = 0.91$ with a standard error 0.08.¹² This indicates that the model does well in both economic terms - $\bar{\beta}$ is close to 1 and far from 0 - and in statistical terms - $\bar{\beta}$ is estimated with a high degree of reliability.

To understand better the importance of sampling error we draw $\lambda(\beta)$ the log likelihood function as a function of β in the left graph below. The solid vertical line in the graph is the left limit of

¹²From Table 1 in the Data Appendix we see that the log-likelihood is equal to $8 \ln(\alpha(ss) + \beta) + 4 \ln(\alpha(s) + \beta) + \ln \alpha(s) + 2 \ln(\alpha(ww) + \beta)$. The solution to the constrained maximization was computed using R (code available).

the 95% confidence interval based on the estimated standard error of $\bar{\beta}$. The graph in the right panel shows the power of the corresponding (one-sided, significance level 2.5%) Wald test showing the test discriminates well against values of β below 0.60.



Because of the non-linearity of the system this is not an entirely accurate guide as to the importance of sampling error. An alternative is to use a likelihood ratio test. Observe that twice log of the likelihood ratio $-2(\lambda(\beta) - \lambda(\bar{\beta}))$ is approximately chi-squared with one degrees of freedom. The cutoff for the 5% upper tail is $\bar{\chi} = 3.841$, so the acceptance region is $-2(\lambda(\beta) - \lambda(\bar{\beta})) \leq \bar{\chi}$ or $\lambda(\beta) \geq \lambda(\bar{\beta}) - \bar{\chi}/2$: the chi.squared line in the left panel above plots $\lambda(\bar{\beta}) - \bar{\chi}/2$ so the the 5% acceptance region is above the line, and rejection region below it. Hence the range [0.60, 0.99] are those values of β that cannot be rejected at the 5% level. The crucial point is that to the left of 0.60 the likelihood drops rapidly implying that $\bar{\beta}$ is a lot different in both economic and statistical terms (despite the limited data) than 0: The variables of outsider and fortification strength do a good job of explaining social organization.

Robustness

To study the robustness of our results we examine three variations on the base model.

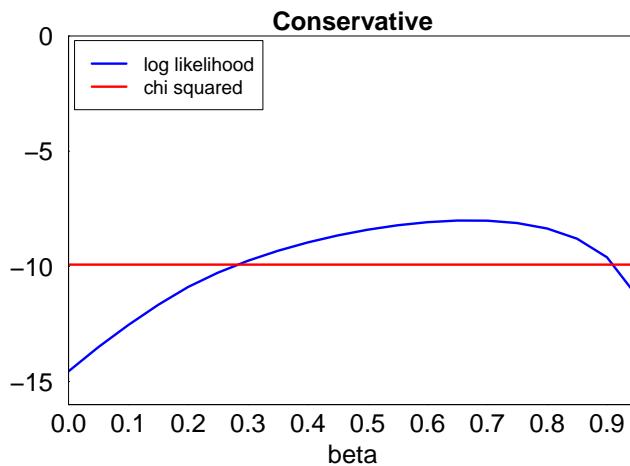
Measurement Error

Measurement error is a serious issue when we deal with data from up to two thousand years ago. Military history and the technology used for sieges is well recorded and does not pose much of an issue. By contrast demographic data especially on peripheral areas is spotty and questionable. There are two main issues with our evaluation of the strength of outsiders: when did outsiders first become strong in China? And did outsiders really become weak in India?

In the case of China we know that the strength of the periphery increased with the arrival of the “five barbarians” from Central Asia causing the fall of the Han Dynasty and the beginning of a period of warring states in 206 CE. This comes well before the Sui/Tung dynasty that begins in 581 CE. But are we correct to classify them as strong between 206 and 581 CE?

In the case of India there is evidence that the population of the subcontinent grew between 1000 and 1500 CE relative to surrounding areas. But are we correct in concluding that the outsiders’ strength was below the threshold in the 16th Century when the Moguls took power? Against the relatively poor demographic data we observe that the Mogul hegemony was short lived and that it fell apart partially because of an invasion from Afghanistan. The situation is complicated because by that time Europeans had arrived on the scene with much better military technology. However, we must recognize the possibility that outsiders were strong even during the period of the Mogul hegemony.

To evaluate the importance of measurement error we take the worst case: we assume that outsiders became strong in China only at the beginning of the Sui/Tung dynasty and that they were always strong in India. This leads to three failures of the theory: the original fact that the model predicts ss for the hegemonic Sui/Tung, plus the additional facts that the model predicts s for the warring states period prior to the Sui/Tung and the fact that the model predicts ss for the hegemonic Mogul period. We reran the estimation: below we plot the likelihood function with the revised data

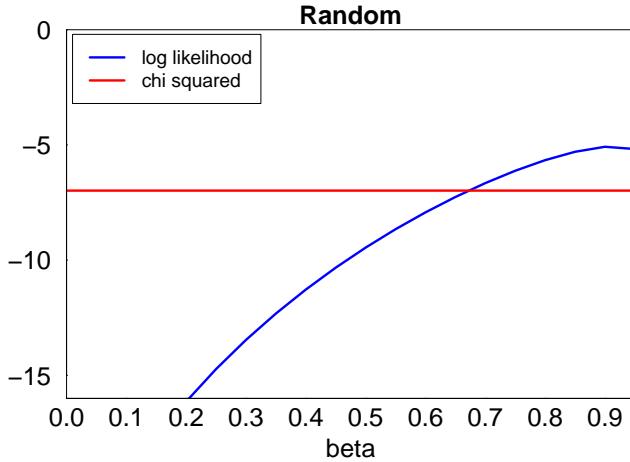


Not surprisingly the theory fares less well - although getting 12 right out of 15 is still not bad. Here the estimate of the slope parameter falls to $\bar{\beta} = 0.67$ and the range generated by the 95%

acceptance region for the likelihood ratio test becomes $[0.21, 0.91]$. While this is shifted substantially left even the lower bound at .21 is still economically very different than 0.

Missing States

One thing the model does get right is that it correctly predicts that the states sw and w will never be observed. An alternative specification of the model is to assume that with probability β the stochastically stable state $\hat{z}(\eta, \varphi)$ is observed and that with probability $1 - \beta$ the observed state is drawn randomly with equal probabilities of $1/5$ over each of the five states. This leads to a slightly higher estimate $\bar{\beta} = 0.92$ and a tighter 95% acceptance region for the likelihood ratio test $[0.68, 0.99]$. Below we plot the likelihood function

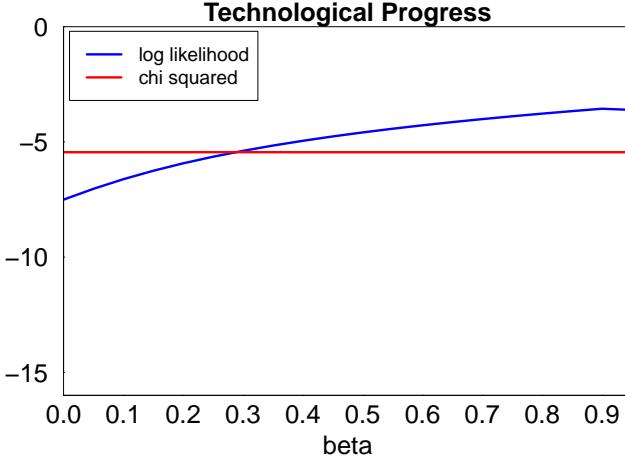


The main difference is in the likelihood ratio against the hypothesis $\beta = 0$. In the base case this was -14.6 , a uniform distribution over states yields the much lower value of -24.1 .

Forecasting Technological Progress

Although the motivation for our model lies in understanding technological progress our model is one of social organization. As indicated in the introduction, we simply accept the hypothesis put forward by others that it is a balance of power that favors technological progress, augmenting this with the observation that to have any hope of explaining any data we need to modify that hypothesis to reflect that it is only an inclusive balance of power that favors technological progress. As an additional check on the model, we can employ that theory. Each observation is assigned an indicator ξ of high (1) or low (0) technological progress and we now take the stochastically stable state to be high technological progress if the social state is ww and one of low technological progress otherwise. We continue to employ a linear probability model, albeit with two rather than three parameters, as there are now two rather than three categories of observed state. The linear conditional probability model is then $\Pr(\xi|\eta, \varphi) = \alpha(\xi) + \beta \cdot 1(\xi = \hat{\xi}(\eta, \varphi))$ where $\hat{\xi}(\eta, \varphi) = 1$ for strong η and weak φ and 0 otherwise.

This leads to the same estimate for the slope as in the base model $\bar{\beta} = 0.92$ but a considerably wider 95% acceptance region for the likelihood ratio test $[0.29, 0.99]$. Below we plot the likelihood function



The key fact that underlies wide range of possible β 's is the fact that one observation is wrong: the Han dynasty in China was an extractive hegemony, but in fact was highly innovative and there was a great deal of technological progress in the economy (see the Data Appendix). This is not a shortcoming of our model, but is something that students of the history of technological change might wish to account for in their models.

4.2. Key Historical Facts

Between China and Europe there are two key differences. First, effective siege technology arrived later in Europe than China, so that an inclusive balance of power developed later: the European Renaissance was some 400 years after the start of the Song dynasty in China. Second, the conquests of Genghis Khan and his successors led to a large outmigration of population from the Mongolian region of China (reversing the earlier outmigration from Central Asia), weakening the outsiders, so that China slipped back into extractive hegemony. The key theoretical point here is that the cannon only helps if outsiders are strong.

While the details of the construction of our data can be found in the Data Appendix, we discuss the key historical points here. In the data there are only two episodes of inclusive societies (both highly innovative, both a balance of power). In the European case it continued, in the Chinese case it died - the historical puzzle ("The Great Divergence") is why the European and Chinese experiences were so different. Our theory says that for an inclusive balance of power two conditions must be met: good siege technology and strong outsiders. Our argument is that for the periods in question both had good siege technology but that in China the Mongolian diaspora weakened the outsiders so much that only extractive hegemony was possible. India during the Moghul period provides some corroborating evidence: they also had cannons but outsiders were weak and again as in China we see an extractive hegemony.

For the key transition in China to the extractive hegemonies following the Song dynasty we should therefore provide evidence for the inclusiveness of Song China and for the Mongolian diaspora, and check that the timing of siege technology and inclusiveness is correct. For example, if the revolution in siege warfare in China was the development of a cannon capable of knocking

down city walls - if this did not happen until after the end of the Song dynasty it could scarcely have played a role in its rise or persistence. While the theory does not require the development of effective siege technology prior to an episode of inclusiveness - according to the theory that could be a random event - for inclusiveness to persist effective siege technology must at least develop contemporaneously. We turn next to the evidence.

Was the Song Dynasty Inclusive?

The Song dynasty from 960 - 1234 BCE is widely celebrated as a golden age of Chinese history - especially in science, engineering, technology, innovation, manufacturing, commerce, and economic prosperity. In contrast to the Renaissance in Europe historians have had less to say about the dawning rights of man during this period. There is substantial evidence, however, that the Song era was unique also in Chinese history for its inclusiveness.

Recall that in our context inclusiveness refers to the relative strength of commercial over military elites. With this in mind we set out criteria for inclusiveness

1. Civilian government: The medieval system in which local military commands are charged with administration and the collection of taxes is evidence of the strength of the military elite. By contrast the absence of such local military units and government by local civilians is evidence of inclusiveness.
2. Mobility. If the base from which political leaders are drawn is broad we judge that to be relatively inclusive.
3. Transfers: How much is transferred to military elites? A key element of our theory is that inclusiveness means the transfers are low, while extractiveness means they are high.

With these criteria in mind, what do we know about the Song and other periods in Chinese history? The basic resource for historical government in China is Finer and Finer (1997). They give detailed information about governance in China in all but the Song period for which we turn to other sources that give explicit comparisons of the Song to earlier and later periods. We should indicate that we are by no means the first to point out the parallels between Song dynasty China and Renaissance Europe: this is the heart of the “Naito Hypothesis” much researched in the Japanese studies of Song China - a good description can be found in Miyakawa (1955). The basic point is that during this period medieval rule by local military leaders was replaced by collaboration between the Emperor and the “masses” - mostly commercial interests to be sure, but also the peasants gained many rights.

Civilian Government. Finer and Finer (1997) indicate that in the non-Song eras the emperor ruled through military commanderies which were charged with local government and fighting “banditry” - which is to say keeping the locals in line. This system declined during the Song period as the Song emperors were deeply suspicious of the military. Military establishments were reduced and military elite lost prominence. A good account can be found in King (1992). During the Song era local control passed from the military and landed aristocracy to a new class - the *Shih*, highly educated literati or gentlemen. These were often those who studied for but failed the imperial exams.

Membership in this class arose from becoming classically educated and was certified by taking the exam. As result the elite expanded to include local magnates, family heads, and informal public servants as well as ex-officials. The commanderies declined in importance and the Shih played a key role in organizing local markets and in the collection and allocation of taxes. Fogel (1984) gives a good account of this. The decline in the power of the political of the military during the Song era is noted by many authors - see also Miyakawa (1955).

Mobility. The primary source of mobility in China has been access to the imperial bureaucracy through the examination system. Although the imperial bureaucracy existed throughout the era Elman (2000) indicates that "Before the Northern Sung, the principal means of entry into the social and political elite was by official recommendation or kinship relations" while Finer and Finer (1997) indicate that after the Song dynasty the examination system became corrupted and eunuchs loyal to the emperor were relatively more important than bureaucrats selected through exams. By contrast during the Song era the imperial examination system reached its peak. Elman (2000), Kracke (1957) and Lee (1985) provide detailed accounts. During the Song many reforms were made to avoid corruption: in particular the grading of exams became anonymous and the exams themselves were recopied before grading. Most civil service positions were filled through the examination system. The exams themselves were relatively democratic as many private schools served to provide training for the exams to local elites. Chaffee (1985) tells us that by the early 1100s the state school system had 1.5 million acres of land that could provide a living for some 200,000 students, and by the end of the dynasty it is estimated (see Ebrey and Walthall (2006)) there were some 400,000 students, more than ten times the number at the beginning of the dynasty. The invention and wide spread use of the printing press also proved a key element to democratizing education.

Transfers. During the Song era there was little or no increase in the size of central government or the military establishment during a period in which the population nearly doubled and there was an explosion in commerce. We have in particular data from Hartwell (1988) on the finances of the Song government: palace expenses (transfers) increased not at all, and a substantial portion of revenues that had been under the control of local military leaders moved instead to the control of the Shih. We can reasonably characterize this as a substantial reduction in relative transfers from the commercial to military elite. After the Song era the military again gained prominence and the central government and military establishment were greatly increased under the Mongols, even in the face of declining population.

*Was the State *ww*?* It is not enough to know that Song China was relatively inclusive: for the state *ww* to hold the same must be true of the rival power. The balance of power lay between Song dynasty China and the competing northeastern Liao/Jurchen state. King (1992) gives a good overview Liao/Jurchen governance. The Liao/Jurchen state was a hybrid state consisting both of northern nomads (the rulers) and Han Chinese. A mixed system of government was employed - one that was relatively more inclusive than the Song as the nomads did not have emperors but

determined the succession through elections. However, the Han Chinese also were incorporated into the Liao court (after about 1011) and the Liao/Jurchen state used civil service exams adopted from those used by the Song. In our view Liao/Jurchen was at least as inclusive as Song China.

The Mongolian Diaspora 1200 - 1300 CE

A key element of our story of why China fell back into extractive hegemony and Europe did not was that the strength of outsiders fell. Mongolian demography is not a widely studied subject, and indeed there are no demographic studies of Mongolia prior to the modern era we have been able to locate. "Mongolian diaspora" is also not a term commonly used by historians. Yet the evidence is plain and straightforward. While we have no count of how many people remained in Mongolia, we have a great deal of data that indicates that most of them left. That is: Genghis Khan and his successors led large armies consisting of the bulk of the male Mongolian population together with their families to conquer wide areas of Asia and Europe. These conquests were successful and the conquerors did not return to Mongolia with loot but settled in to rule their conquered territories. Although generally viewed as warfare, Genghis Khan's conquests were also a demographic event - reversing in many ways the earlier diaspora from Central Asia.

When and where the Mongols conquered is not a subject of great dispute. When Genghis Khan's empire was divided, it was divided into four regions. The Great Khanate constituted China and whoever remained in Mongolia. The Chagatai Khanate constituted most of Central Asia; the Golden Horde were based in Eastern Russia but ruled as well over most of the Kiev region. The Ilkhanate constituted Persia and the surrounding areas. The point of course is that these are large areas and not particularly close to Mongolia.

Aside from the fact it would have been difficult to maintain rule over such large areas for centuries if all the Mongolian soldiers returned to Mongolia, we have direct evidence that the Mongolians settled where they conquered. We find, first, that where the Mongol men went so did the Mongol women: see for example De Nicola (2017). Moreover, there are accounts of what happened to the Mongol conquerors:

Once China was taken over, the Mongol garrison troops had to get their livelihood from their own agriculture and that of their slaves on the depopulated lands allotted to them in North China. The fighting capacity of their hereditary military households soon deteriorated. Mongol officers formed a segregated and self-perpetuating salaried aristocracy - the superior military wing of the imperial bureaucracy - but in general the Mongol soldiery in China became impoverished. ...many lost their lands, even had to sell their families, sometimes absconded and became vagrants. To be hereditary soldiers in peacetime turned out to be a disaster. King (1992)

Siege Technology and Inclusiveness

In the model we have studied timing is not irrelevant. If an inclusive balance of power requires weak fortifications - or what amounts to the same thing - strong siege technology, that siege

technology must be contemporaneous with the inclusive society. There is no point that arguing the developments in siege technology that took place long before or long after inclusiveness are somehow correlated with it. Here we briefly examine the history and timing of siege technology.

Every tourist has noticed that a feature of ancient cities absent from modern cities are the existence of walls. There is archaeological evidence of brick city walls as early as 2500 BCE (Fletcher and Cruikshank (1996)). Since that time there has been a race between the technology of walls and the technology of siege: better design and construction techniques for walls have competed with the development of battering rams, ladders, siege towers, ballistas and catapults. With the development of the gunpowder the technology of walls gradually became obsolete: city walls today would be completely useless for defensive purposes as modern artillery would reduce them to ruin in a matter of moments. In World War II the immense and sophisticated Belgium fortress of Eben-Emael was rendered ineffective by a German force of 75 men in a few hours (Kauffmann and Jurga (2002)).

China. Gunpowder was invented in China sometime before 808 CE (Unknown (808)). Early in the Song period it began to play an increasingly important role in Chinese warfare. A military manual (Gonglang, Du and Weide (1044)) from 1044 CE describes a variety of gunpowder weapons that were widely used. What proved most important in practice was the development of the bomb. Siege technology prior to gunpowder involved the use of catapults and ballistas of various types to hurl stones at or over city walls. Replacing stones with bombs proved extremely effective. In the 1100-1200s during the various wars between the Song and Liao/Jurchin there was an R&D race with each side vying to develop improved explosives and each holding a temporary military edge over the other as they developed larger and more explosive bombs. Large bomb factories were built and bombs produced in large quantities by both sides and we read, for example, descriptions of armies as having three thousand men and three thousand thunderclap bombs (Andrade (2016)).

Ancient sieges had been chancy because a besieged city, with its stored supplies, often could outlast the besiegers foraging in the barren countryside. The new Song weapons now could batter walls and gates, explode gunpowder mines, and light fires within the walls. King (1992)

Thus the evidence that the revolution in siege warfare was contemporaneous with the Song era is strong.

Europe. Gunpowder technology did not reach Europe until some 400 years after it was invented in China - most likely brought by the Mongols. The first hard evidence we have is that of Bacon (1276) from 1276 who describes the manufacture and use of gunpowder. By 1346 cannons were used in the siege of Calais and in the 1375 siege of Saint-Sauveur-le-Vicomte the French breached the fortress walls with guns weighing over 1 ton and firing 50 kg stone balls. The fall of Constantinople

to a handful of cannons in 1453 marked the end of the era of the castle and the fort.¹³

We know that Europe during the Renaissance flowered with the disappearance of medieval fiefdoms and the development of relatively inclusive societies in Italy, the Netherlands and the Hanseatic area. When did this happen relative to the revolution of siege warfare? We note that the fall of Constantinople was itself an important event intellectually in the sense that it triggered the exodus of Byzantine scholars to Italy where they brought with them Greek ideas, including political ideas such as democracy. However, unlike Song China where we have only anecdotal evidence of inclusiveness, in Europe there are two datasets that indicate the timing of the spread of inclusiveness. The first is from Serafinelli and Tabellini (2017) who document the rise of the Comune - charters granting urban areas substantial power of self-governance. In 1000 CE there are no Comune. In 1100 about 10% of urban areas are Comune. This rises to 55% in 1200 and 65% by 1500. The second is from Blank, Dincecco and Zhukov (2017) who document in which European countries the parliament had a voice over taxes. In 1200 this consisted of one country, Sicily, under the Normans. By 1400 the parliaments of Austria, Bohemia, Castille, France, Poland, Portugal and Sicily as well as many smaller states had a voice over taxes. We somewhat arbitrarily chose the date of the publication of the Ordinances of Justice in Florence in 1293 CE as an important political event marking the beginning of the era of greater inclusiveness.

Note that the rise of the inclusiveness began before the advent of effective siege warfare and continued to rise after. The theory, however, does not say “cannons cause inclusiveness.” It says “without the advent of the cannon the inclusiveness that began earlier was unlikely to persist.”

Was the State ww?. There were throughout the era in question often many more than two societies in Europe. However, the contest between great power after the fall of Rome was largely between the Holy Roman Empire - basically the German speaking peoples - and France. During the Renaissance when inclusiveness was increasing we see from Blank, Dincecco and Zhukov (2017) account of parliamentary power that indeed both sides: France on the one side, and especially Austria and Bohemia on the other did indeed become more inclusive.

5. Discussion

Early History

Although our focus is on the Common Era - for the obvious reason of data availability - the BCE history of Egypt, China and Persia is of interest. In this era prior to early globalization great hegemonies spanned centuries.

In Egypt during the 1,617 years from 2686 BCE to the end of the new Kingdom in 1069 BCE there are roughly 500 years of “intermediate period” during which there is not an extractive hegemony (Shaw (2000)). It is also the case that the decline of Egypt coincides with the arrival of

¹³See, for example, Sumption (1999) for an account of the use of cannons in siege warfare during the hundred years war and Philippides and Hanak (2011) for their use during the siege of Constantinople.

the historically mysterious sea people in roughly 1200-900 BCE. They may or may not have been successful invaders: our theory indicates they need not have been - the strengthening of outsiders is in general inimical to hegemony.

In China during the 1820 years from 1600 BCE to 220 CE there are 501 years of Spring and Autumn and warring states period - the remaining being extractive hegemony Wright (2006). In Persia during the 1,201 years from 550 BCE to 651 CE we have extractive hegemony except for about 87 years between the Achaemenid Empire and Parthian/Sassanid Empires Daryae (2012).

The basic point is that Before the Common Era when outsiders were less strong hegemonies were indeed more frequent.

The Weak and the Strong

As we observed, the extractive advantage is positive. How can we justify this conclusion that inclusive societies are weaker than extractive ones in light of a modern literature (see, for example, Reiter and Stam (2002)) arguing that democracies are more successful than autocracies at conflict, at least in the last several centuries? First we observe that “strength” in the way we measure it here means “prevails more frequently” and this is driven in the model by stronger incentives to initiate conflict not a greater ability on the battlefield. Indeed, if the parameters of the conflict resolution function favor the defense sufficiently, then in a head to head competition at sw it will be the case that w , being most often the defender, will have a higher success rate than s , which is most often the attacker. This idea is consistent with the modern literature: Reiter and Stam (2002) attribute the greater success of democracies to greater selectivity in the wars that they fight. Second: the greater success rate is controversial. Desch (2008) argues that using data over a longer period of time (several millennia rather than centuries - more relevant to the period we consider) there is no particular advantage of inclusive societies, and that it is material advantage that matters (our assumption). This is backed up by Biddle and Long (2004) who analyze data on battles in the last hundred years showing that when factors such as material strength is accounted for democracies actually do less well in battle than autocracies.

There is potentially a second form of extractive advantage: a society politically dominated by military elites may find it less costly to raise resources for conflict. Specifically we might suppose that in the cost of effort provision $C(x_i) = (\gamma/2)x_i^2$ instead of a common cost factor γ we have cost factors γ_s, γ_w with $\gamma_w > \gamma_s$. We analyze this case in the Web Appendix. In the crucial case where $\tau = 0, \varphi = 0$ we find that it remains the case the modified radius of ww is greater than that of ss provided that γ_w is not too large. Specifically in this case we show that $R(ww) > R(ss)$ if (and only) if $\gamma_w < 3\gamma_s$.

The Home Field Advantage

Although it plays no role in our historical analysis, there is a feature of our model worth pointing out: using the fact that $u(0, y_d, \varphi) = 0$ we have $r(ww \rightarrow w) - r(ss \rightarrow s) = \Pi_b(\eta) + u(1, 1, \varphi) > 0$. This says the strong perform well on foreign ground while the weak do not - that is in attacking a strong power the strong do better than the weak do in attacking a weak power. It also embodies

the idea of “home field advantage” particularly for the weak. The idea that defending home turf is an advantage is an important feature of this model that differs from our earlier work: we briefly indicate how this model differs from those earlier models. Levine and Modica (2013) and the example of China in Levine and Modica (2016) use a common framework in which there are many units of land and the weaker society always has zero resistance. Here we have simplified by having only a single unit of land belonging to each society - but the same point as we make here could be made in the many units of land case with a border: the weaker power can still have zero resistance - but which power is weaker changes when the border is crossed due to the disadvantage of fighting on enemy territory - an effect that we believe is important but is absent in our earlier work. Here - unlike that earlier work - resistance is endogenously brought about by the decisions of groups and not an exogenous “state power” similar to the notion of “state capacity” used by Besley and Persson (2010). In subsequent work Levine and Modica (2017), also using the many units of land framework, but restricting attention to a single type of society, the intervention of outsiders acted like a border. For the type of conflict studied in that paper where modern societies both have claims to the same land that assumption still seems to us appropriate, but for the situation considered here of evolution between societies which may conquer one another the notion of a border seems most appropriate.

The idea that the strong perform well whether they are fighting “at home” or “away” while the weak perform well only when they are fighting “at home” arises in this model because the commercial elites have little reason to attack but much reason to defend. The fact that the strong do well both at home and away while the weak only do well at home is common phenomenon well known to sports fans. For example, we gathered data from NBA basketball¹⁴ and found that when weak teams play each other the visiting team won about half as often as when strong teams play each other.

Robustness: Linear Quadratic Technology

While the basic model is highly stylized the results in theorem 2 are robust to many details. We discuss several extensions here.

With respect to the specific linear/quadratic conflict resolution function we observe that theorem 2 relies only on the conclusion of theorem 2: any conflict technology that results in a $u(y_a, y_d, \varphi)$ function satisfying the qualitative properties of that theorem yield our main theorem, theorem 1. Certainly small perturbations from the linear quadratic model will do so. In fact these conclusions are relatively robust. Consider the more general conflict resolution function $\pi(x_a, x_d) = \Pi_\zeta(\eta) + \alpha\pi_0(x_a, x_b, \varphi)$ where π_0 is strictly increasing in x_a and decreasing in x_b , strictly so for $\varphi < 1$ and independent of x_b for $\varphi = 1$ together with convex effort cost $c(x_a)$. If, as in the base model, the outcome of π_0 is uncertain enough and $c(x_a)$ is convex enough existing results (see Herrera, Morelli

¹⁴Using data for the 2016 NBA season from http://www.espn.com/nba/team/schedule/_/name/gs/year/2017 we computed the records of the three best teams in the Western Conference against each other and the same for the three worst teams.

and Nunnari (2015)) imply the existence of a unique interior pure strategy equilibrium. Defining $u(y_a, y_d, \varphi) = \alpha\pi_0(\hat{x}_a, \hat{x}_b)y_a - c(\hat{x}_a)$ then certainly $\Pi_\zeta(\eta)y_a + u(y_a, y_d, \varphi)$ is non-negative, less than or equal to one and under mild conditions the intuitive conditions that it is strictly increasing in y_a and that $u(y_a, y_d, \varphi)$ is strictly decreasing in y_d , and satisfies $u(0, y_d, \varphi) = 0$ hold. The key qualitative features of the model require also that $v(\varphi) = u(1, 1 - \tau, \varphi) - u(\tau, 1, \varphi) - u(1 - \tau, 0, \varphi)$ is negative for small τ and φ and positive for large φ . Consider first that for $\tau = 0$ and $\varphi < 1$ we have $v(\varphi) \equiv u(1, 1, \varphi) - u(1, 0, \varphi) < 0$ by monotonicity, so indeed $v(\varphi)$ is negative for small τ . For large φ consider $v(1) = u(1, 1 - \tau, 1) - u(\tau, 1, 1) - u(1 - \tau, 0, 1) = u(1, 0, 1) - u(\tau, 0, 1) - [u(1 - \tau, 0, 1) - u(0, 0, 1)]$ so that $v(1) > 0$ if $u(y_a, 0, 1)$ is strictly convex in y_a . To see why this must be the case, notice that when $\varphi = 1$ defensive effort does not matter, the defender optimally chooses $x_d = 0$ and the aggressor faces the simple problem of maximizing $(\Pi_\zeta(\eta) + \alpha\pi_0(x_a, 0, 1))y_a - c(y_a)$. The first and second order conditions for the optimum together with the implicit function theorem show that the solution of this problem is a convex function of y_a and therefore $u(y_a, 0, 1)$ is convex as well.

Robustness and Time

In the model is that conflict is rare and the expected time between conflicts long. This has two consequences. First, the random choice of active group does not much matter. It might be that under hegemony the prospective rebels draw two independent utility shocks one for inclusive and one for extractive institutions - if the larger of the two resulting utilities is positive then a rebellion attempting to install the “more profitable” institutions is launched. However: since the probability either is positive is very small the chance that both are simultaneously positive is negligible and we might as well assume one is chosen at random as a prospective agenda. Similarly in a balance of power each society may simultaneously draw independent utility shocks: again the chance both are positive simultaneously is negligible. A more technical way to say this is that the resistance to a single positive shock is much less than the resistance to a simultaneous shock, so least resistance transitions involve only a single positive shock.

The other consequence of conflict being rare is that the model is robust to the assumption that groups are myopic. If we interpret the utility shocks as the long-term present value of the effect of conflict (death, disruption of social organizations, destruction of capital, political popularity and so forth) then groups may as well discount the future with a discount factor $\delta < 1$. With high probability another “good” draw of the utility shock will be a long time in coming so that in effect the decision is a static one: stay with the status quo for a long time, or attempt to switch to an alternative which if successful will also last a long time.

In a similar vein: we might wonder whether occupier military elites might not choose to collect transfers lower than the most that they can extract in hopes of reducing the chance of rebellion. Since the chances of rebellion are very small and the transfer reduction would have to be permanent this would not make sense.

Who are the Rebels?

Continuity considerations imply that the model is robust with respect to perturbations that have a small effect on resistances.¹⁵ For example, while it makes sense that in an inclusive society occupier commercial elites take the decisions and in an extractive state occupier military elites do, for the occupied society the situation is less clear cut. Let y_m and y_e denote the utility gain in a hegemony to the occupied commercial elites and occupied military elite from a rebellion (with respect to some already determined goal). The basic model assumes that if the hegemony is inclusive only the incentives of the commercial elites y_m matter, while if the hegemony is extractive only the incentives of the military elites y_e matter. Consider instead a model where in an inclusive hegemony a weighted average $(1 - \omega)y_m + \omega y_e$ of the gain to each group represents the “overall gain” to rebellion, and likewise in an extractive hegemony $\omega y_m + (1 - \omega)y_e$ represents the “overall gain.” Simple continuity considerations show that the qualitative results of theorem 1 are robust to ω slightly positive. In fact ω can be quite large: instead of assuming $\omega = 0$ assume only that $\omega < 1/2$ to reflect that in an inclusive society commercial elites do have greater political strength than military elites and vice versa. In the Web Appendix we show that the qualitative conclusions of the model are robust to this perturbation. The structure of circuits remains unchanged and it remains the case that only s, ss and ww can be stochastically stable. The cutoff for an extractive hegemony over an extractive balance of power $\eta^*(\varphi, \omega)$ now depends on the military technology and on ω , and the specific numeric value of the cutoff τ^* changes: but we show that when $\tau = 0$ and $\varphi > 0$ then ww has a greater modified radius than ss and that when $\varphi = 0$ and $\tau > 0$ then ss has greater modified radius than ww .

This is an appropriate place to comment on an important driving force in the model: that $r(w \rightarrow sw) = U - \Pi_h(\eta) - u(0, 0, \varphi)$ and $u(0, 0, \varphi) = 0$ implying very high resistance to moving from an inclusive hegemony to a balance of power where rebellion has led to extractive institutions. In the basic model this is due to the fact that political power among the rebels lies with the commercial elites who have no reason to replace extractive foreign institutions with extractive domestic institutions ($y_a = 0$). It might seem that the commercial elites being indifferent and the military elites standing to gain 1 by rebellion the commercial elites might go along with the military elites so that in fact $y_a = 1$. Our analysis of $\omega < 1/2$ shows that this is not true. Why not? The point is that $y_a = 0$ does *not* imply the commercial elites are indifferent. Their utility gain is 0 *plus* the utility shock which is with high probability very negative. If we assume that a common utility shock is drawn by the commercial elites and the military elites then because the probability of larger shocks drops very rapidly the most likely scenario in which the military elites wish to rebel is one in which they have only a slight preference for doing so. On the other hand a utility shock that leaves the military elites slightly preferring to rebel will have the commercial elites rather more strongly preferring not to rebel: it is not the cases that the commercial elites are indifferent

¹⁵Perturbations such as a random determination of whether commercial elites or military elites are in control that have a discontinuous effect on resistances and lead in general to entirely different conclusions.

so should defer to the wishes of the military elites, rather the opposite. Our analysis in the web appendix of the full model shows, however, that this is not enough: some slight political advantage must lie with those favored by the dominant institutions to generate the qualitative features of the basic model.

6. Conclusion

In this paper we develop a simple stylized model of conflict between societies. There are two societies contending over resources and one may or may not rule over the other. There are two groups in each society: commercial elites and military elites. Institutions can be either inclusive or extractive with inclusive institutions substantially more favorable for the commercial elites. We examine stochastic stability and show that when outsiders are weak extractive hegemonies will predominate. When outsiders are strong military technology matters: if outcomes are insensitive to defensive strength - fortifications are well able to resist siege - an extractive balance of power will predominate, while good siege technology (the cannon in particular) will result in the predominance of an inclusive balance of power. This, as is widely agreed, is the configuration more favorable to innovation and economic development.

We apply this theory historically. We argue that generally speaking the importance of outsiders has increased over time and siege technology has improved. We document that the increased importance of outsiders in Europe and India preceded the advent of good siege technology: our theory implies that we should see an extractive hegemony followed by an extractive balance of power followed by an inclusive balance of power. In Europe this corresponds roughly to the Roman Empire, followed by the early medieval period followed by the Renaissance. In India we argue that outsiders were strong and that good siege technology arrived “too late” so that in India there was generally an extractive balance of power. In China we see a back and forth between extractive hegemony and extractive balance of power until the relatively early arrival of good siege technology coincides with the flowering of the Song and rival states. However a striking demographic event - the Mongolian diaspora - greatly weakened the strength of outsiders resulting in a reversion to extractive hegemony from the Yuan dynasty onward.

The notion that the growth of modern inclusive societies was driven by the “democratization” of warfare is not a new one; nor, of course is the idea that outsiders destroy hegemonies. Our account is a more subtle one: we view “democratization” of warfare as being driven by the decline of the fortress because of the cannon rather than the introduction of hand held fire-arms. We also see the impact of outsiders not so much in conquering empires but in weakening their resistance to revolution and strengthening the balance of power. The theory appears to do a reasonably good job of organizing the historical data.

References

- Acemoglu, Daron and James A. Robinson (2012): *Why Nations Fail: The Origins of Power, Prosperity, and Poverty*, Crown Publishing Group.
- Andrade, Tonio (2016): *The Gunpowder Age: China, Military Innovation, and the Rise of the West in World History*, Princeton University Press.
- Baburnama, AC, I Habib and KM Baipakov, eds. (2004): *History of Civilizations of Central Asia: Development in Contrast : from the Sixteenth to the Mid-Nineteenth Century*, United Nations Educational.
- Bacon, Roger (1276): *Opus Majus*.
- Basu, Dipak, Victoria Miroshnik (2017): *India as an Organization: Volume One: A Strategic Risk Analysis of Ideals, Heritage and Vision*, Springer.
- Besley, Timothy and Torsten Persson (2010): "State Capacity, Conflict and Development," *Econometrica* 78: 1-34.
- Biddle, S. and S. Long (2004): "Democracy and military effectiveness: A deeper look," *Journal of Conflict Resolution* 48: 525-546.
- Bisin, A., and T. Verdier (2001): "The economics of cultural transmission and the dynamics of preferences," *Journal of Economic Theory*, 97: 298-319.
- Bisin, A., and T. Verdier (2005): "Cultural Transmission," *The New Palgrave Dictionary of Economics*.
- Bowles, Samuel and Jung-Kyoo Choi (2013): "Coevolution of farming and private property during the early Holocene" *Proceedings of the National Academy of Science*, doi:10.1073/pnas.1212149110
- Blank, Meredith, Mark Dincecco, and Yuri M. Zhukov (2017): "Political regime type and warfare: evidence from 600 years of European History," University of Michigan.
- Choi, Jung-Kyoo and Samuel Bowles (2007): "The coevolution of parochial altruism and war," *Science* 318: 636-40.
- Cipolla, Carlo (1965): *Guns, Sails, and Empires: Technological Innovation and the Early Phases of European Expansion, 1400- 1700*.
- Chaffee, John W. (1985): *The Thorny Gates of Learning in Sung China*, Cambridge University Press
- Daryaee, Touraj (ed)(2012): *Persia Oxford Handbook of Iranian History*, Oxford University Press.
- De Nicola, B. (2017): *The Study of Women in the Mongol Empire*, Edinburgh University Press.
- Desch, Michael Charles (2008): *Power and military effectiveness: the fallacy of democratic triumphalism*, Johns Hopkins University Press.
- Diamond, Jared (1998): *Guns, Germs, and Steel: The Fates of Human Societies*.
- Dutta, R., D. K. Levine, N. W. Papageorge, and L. Wu (2018): "Entertaining Malthus: Bread, Circuses, and Economic Growth," *Economic Inquiry*, 56(1), 358-380.
- Ebrey, Patricia Buckley and Anne Walthall (2006): *East Asia: A Cultural, Social, and Political History*, Houghton Mifflin Company.
- Ellison, Glenn (2000): "Basins of Attraction, Long Run Stochastic Stability and the Speed of Step-by-step Evolution," *Review of Economic Studies* 67: 17-45.
- Elman, Benjamin A (2000): *A Cultural History of Civil Examinations in Late Imperial China*, University of California Press.
- Finer, S.E. and Finer, S.E. (1997): *The History of Government from the Earliest Times*, three volumes, Oxford University Press.
- Fletcher, Banister and Cruickshank, Dan (1996): *Sir Banister Fletcher's A History of Architecture* (20th ed.). Architectural Press.

- Fogel, Joshua (1984): *Politics and Sinology: The Case of Naita Konan (1866–1934)*, Council on East Asian Studies, Harvard University.
- Garfinkel, Michelle and Stergios Skaperdas (2007): "Economics of Conflict: An Overview," *Handbook of Defense Economics*, Elsevier.
- Gongliang, Zeng, Ding Du and Yang Weide (1044): *Wujing Zongyao*.
- Greif, A., and G. Tabellini, G. (2010): "Cultural and institutional bifurcation: China and Europe compared," *American Economic Review*, 100: 135-40.
- Hartwell, Robert M. (1988): "The Imperial Treasuries: Finance and Power in Song China," *Bulletin of Sung and Yüan Studies* 20: 18-89.
- Hausken, Kjell (2005): "Production and Conflict Models Versus Rent Seeking Models," *Public Choice* 123: 59-93.
- Hayek, F. A. (1944): *The Road to Serfdom*, Dymock's Book Arcade.
- Herrera, H., M. Morelli, and S. Nunnari (2015): "Turnout Across Democracies," *American Journal of Political Science*.
- Hirshleifer, Jack (2001): *The dark side of the force: economic foundations of conflict theory*, Cambridge University Press.
- Hoffman, Philip T. (2013): *Why Did Europe Conquer the World?* forthcoming Princeton University Press.
- Kauffmann, J.E. and R. Jurga (2002): *Fortress Europe: European Fortifications of World War II*, Da Capo Press, USA.
- Kandori, Michihiko, George Mailath, and Rafael Rob (1993): "Learning, Mutation, and Long Run Equilibria in Games," *Econometrica* 61: 29-56.
- King, John (1992): *China: A New History*, Belknap Press/Harvard University Press.
- Kracke, E. A., Jr. (1957): *Region, Family, and Individual in the Chinese Examination System*, Fairbank,
- Landes, D. S. (2003): *The unbound Prometheus: technological change and industrial development in Western Europe from 1750 to the present*, Cambridge University Press.
- Lee, Thomas H.C. (1985): *Government Education and Examinations in Sung [Song] China*, Chinese University Press/St. Martin's Press.
- Levine, David and Salvatore Modica (2012): "Conflict and the Evolution of Societies," working paper, www.dklevine.com.
- Levine, David and Salvatore Modica (2013): "Anti-Malthus: Conflict and the Evolution of Societies," *Research in Economics* 67: 289-306
- Levine, David and Salvatore Modica (2016): "Dynamics in stochastic evolutionary models", *Theoretical Economics* 11: 89-131
- Levine, David and Salvatore Modice (2017) "An Evolutionary Model of Intervention and Peace", working paper www.dklevine.com
- Lin, Justin (January 1995). "The Needham Puzzle: Why the Industrial Revolution Did Not Originate in China". *Economic Development and Cultural Change*. 43 (2): 269–292
- Liu, Yingqui; Chunjiang Liu (2007). "Diagnosing The Cause of Scientific Standstill, Unravelling The Needham Puzzle". *China Economist*. 10: 83–96.
- Maddison, Angus (1998): *Chinese Economic Performance in the Long-Run*, OECD Development Centre, Paris.
- Maddison, Angus (2013): *Historical Statistics for the World Economy: 1-2003 AD*, spreadsheet available at www.gdcc.net.
- McNeil, William (1963): *The Rise of the West*, University of Chicago Press.
- Miyakawa, Hisayuki (1955): "An Outline of the Naito Hypothesis and Its Effects on Japanese Studies of China," *The Far Eastern Quarterly* 14: 533-552.

- Mokyr, J. (2002): *The gifts of Athena: Historical origins of the knowledge economy*, Princeton University Press.
- Morris, I. (2010): *Why the west rules - for now: The patterns of history and what they reveal about the future*, Profile books.
- Olson, M. (1982): *The rise and decline of nations: Economic growth, stagflation, and social rigidities*, Yale University Press.
- Philippides, M. and Hanak, W.K. (2011): *The siege and the fall of Constantinople in 1453: historiography, topography, and military studies*, Ashgate Publishing.
- Reiter, D., and A.C. Stam (2002): *Democracies at war*, Princeton University Press.
- Rogers D. S., O. Deshpande and M. W. Feldman (2011): “The Spread of Inequality,” PLoS ONE 6.
- Rosenthal, J. L., and R. B. Wong, R. B. (2011), *Before and Beyond Divergence*, Harvard University Press.
- Saturnino, Monteiro (2011): *Portuguese Sea Battles Volume I – The First World Sea Power*
- Serafinelli, M., and Tabellini, G. (2017), “Creativity over time and space,” working paper, Bocconi.
- Shaw, Ian (ed) (2000): *The Oxford History of Ancient Egypt*: Oxford University Press.
- Sheehan, Michael J. (1996): *The Balance of Power: History and Theory*, Taylor and Francis.
- Sumption, J. (1999): *The Hundred Years War, Volume 1: Trial by Battle*, University of Pennsylvania Press.
- unknown (808): *Taishang Shengzu Jindan Miju*.
- Wright, Edmund (2006): *A Dictionary of World History*, second edition, Oxford University Press.
- Young, Peyton (1993): “The Evolution of Conventions,” *Econometrica* 61: 57-84.

Appendix: The Conflict Subgame

Theorem. [Theorem 2 in the text] The conflict subgame has a unique Nash equilibrium independent of the state and strength of outsiders. The total utility gain to the aggressor $\Pi_\zeta(\eta)y_a + u(y_a, y_d, \varphi)$ is non-negative, less than or equal to one and strictly increasing in y_a . The partial utility gain $u(y_a, y_d, \varphi)$ is strictly decreasing in φ , decreasing in y_d , and satisfies $u(0, y_d, \varphi) = 0$ and $u(y_a, y_d, 1) = u(y_a, 0)$. There is a $0 < \tau^* < 1$ and a strictly decreasing function $0 < \varphi_\tau < 1$ such that the function $v(\varphi) \equiv u(1, 1 - \tau, \varphi) - u(\tau, 1, \varphi) - u(1 - \tau, 0, \varphi)$ satisfies $|v(\varphi)| < \alpha$ with $v(\varphi) < 0$ for $\tau < \tau^*$ and $\varphi < \varphi_\tau$ and $v(\varphi) > 0$ otherwise.

Proof. Recall that $0 \leq x_a, x_d \leq 1$, $0 \leq y_a, y_d \leq 1$. The objective functions for the aggressor and for the defender are respectively

$$\begin{aligned} & \Pi_\zeta(\eta)y_a + \alpha(-\varphi + (x_a - (1 - \varphi)x_d))y_a - (\gamma/2)x_a^2 \\ \text{and } & [1 - \Pi_\zeta(\eta) - \alpha(-\varphi + (x_a - (1 - \varphi)x_d))]y_d - (\gamma/2)x_d^2. \end{aligned}$$

In this simple linear quadratic model the optimal choice for aggressor and defender are independent of each other, so the equilibrium is certainly unique. Set $\beta = \alpha/\gamma$. The optimum for the aggressor is $\hat{x}_a = \beta y_a$ and for the defender is $\hat{x}_d = \beta(1 - \varphi)y_d$. Observe that we certainly have $\alpha < 1/2$ while $\gamma \geq 1$ so that $\beta < 1/2$ and these are interior solutions. Hence the partial utility gain to the aggressor is given by

$$\begin{aligned} u(y_a, y_d, \varphi) &= \alpha(-\varphi + (\beta y_a - \beta(1 - \varphi)^2 y_d))y_a - (\gamma/2)\beta^2 y_a^2 \\ &= \alpha[(-\varphi - \beta(1 - \varphi)^2 y_d)y_a + (1/2)\beta y_a^2]. \end{aligned}$$

This is clearly strictly decreasing in y_d and satisfies and $u(y_a, y_d, 1) = u(y_a, 0)$; and its derivative with respect to φ is proportional to $2\beta(1 - \varphi)y_d - 1$ which is negative since $\beta < 1/2$ and $y_d \leq 1$. Observe that the total utility gain is $[\Pi_\zeta(\eta) - \alpha(\varphi + \beta(1 - \varphi)^2 y_d)]y_a + (\alpha/2)\beta y_a^2$ and that $\Pi_\zeta(\eta) - \alpha(\varphi + \beta(1 - \varphi)^2 y_d) \geq \Pi_\zeta(\eta) - \alpha > 0$ by assumption. Hence the total utility gain is non-negative and strictly increasing in y_a .

We next observe that $u(y_a, y_d, \varphi)$ is decreasing in y_d and satisfies $u(0, y_d, \varphi) = 0$. We compute $v(\varphi) \equiv u(1, 1 - \tau, \varphi) - u(\tau, 1, \varphi) - u(1 - \tau, 0, \varphi)$. The computation gives

$$v(\varphi) = \alpha\beta((1 - \varphi)^2(2\tau - 1) + (1 - \tau)\tau).$$

Hence $-\alpha \leq v(\varphi) \leq \alpha(-\tau^2 + 3\tau - 1) \leq \alpha$ for $0 \leq \tau \leq 1$. If $2\tau - 1 > 0$ clearly $v(\varphi) > 0$ for all φ . For $2\tau - 1 < 0$ the function $v(\varphi)$ is increasing in φ and we may solve $v(\varphi_\tau) = 0$ to find

$$\varphi_\tau \equiv 1 - \sqrt{\frac{(1 - \tau)\tau}{1 - 2\tau}}.$$

This is obviously strictly less than 1. It is positive if $q(\tau) = 1 - 2\tau - (1 - \tau)\tau > 0$. From the

quadratic equation we find $q(\tau^*) = 0$ for $\tau^* \approx 0.38 < 1/2$. For $\tau > \tau^*$ we have $q(\tau) < 0$ hence $v(\varphi) > 0$ for all φ . When $\tau < \tau^*$ then $0 < \varphi_\tau < 1$ and for $\varphi > \varphi_\tau$ we have $v(\varphi) > 0$ while for $\varphi < \varphi_\tau$ we have $v(\varphi) < 0$. Finally, we compute $\frac{d\varphi_\tau}{d\tau} \propto -\frac{(1-2\tau)^2 + 2(1-\tau)\tau}{2(1-2\tau)} < 0$. \square

Data Appendix

Data

There are two exogenous variables, the strength of outsiders η and of fortifications φ . The endogenous variable is the state z . Table 1 gives the data.

Table 1: Data. The duration of most episodes is from 2.2 to 5.6 centuries.

region	description	period	duration	outsiders η	fortifications φ	actual state	stochastically stable state	technological progress
India	Classical/Medieval	200 BCE - 320	5.2	strong	strong	<i>ss</i>	<i>ss</i>	0
India	Classical	321 - 650	3.3	strong	strong	<i>ss</i>	<i>ss</i>	0
India	Early Medieval	651-1200	5.5	strong	strong	<i>ss</i>	<i>ss</i>	0
India	Late Medieval	1200 - 1525	3.3	strong	strong	<i>ss</i>	<i>ss</i>	0
India	Mogul	1526 - 1748	2.2	weak	weak	<i>s</i>	<i>s</i>	0
China	Han	202 BCE - 220	4.2	weak	strong	<i>s</i>	<i>s</i>	1
China	Warring Kingdoms	221-588	3.7	strong	strong	<i>ss</i>	<i>ss</i>	0
China	Sui/Tang	589 - 906	3.2	strong	strong	<i>s</i>	<i>ss</i>	0
China	Song	960 - 1279	3.2	strong	weak	<i>ww</i>	<i>ww</i>	1
China	Yuan/Ming/Qing	1280-1839	5.6	weak	weak	<i>s</i>	<i>s</i>	0
Europe	Roman	149 BCE - 329	4.8	weak	strong	<i>s</i>	<i>s</i>	0
Europe	Byzantine	330 - 628	3.0	strong	strong	<i>ss</i>	<i>ss</i>	0
Europe	Medieval	629 - 1054	4.3	strong	strong	<i>ss</i>	<i>ss</i>	0
Europe	Middle Ages	1054 - 1292	2.4	strong	strong	<i>ss</i>	<i>ss</i>	0
Europe	Renaissance	1293-1607	3.1	strong	weak	<i>ww</i>	<i>ww</i>	1

Regions

There is not much doubt about the great historical centers of civilization being Europe (including the Middle East), China and India. According to the model the core should have limited geographical obstacles to conquest and hegemony should be feasible given the military technology of the time. Hence within the core we should see at least some time hegemony, and certainly not see regions that remain immune from conquest over long periods of time. By contrast the periphery should be historically immune from invasion, while have a demonstrated capability of military intervention in the core: there should be evidence of a one-way geographical barrier.

Our primary sources are historical maps. We examine the history of each of the regions.

India

We take the core area of India to be the area occupied by the British during the Raj and the periphery to be Central Asia. Specifically the boundaries of the core are the Himalayas in the North, the jungles of Myanmar in the East, and Afghanistan in the West.

Period. Our start date is 200 BCE which is considered as the beginning of the “classical” period of Indian history which overlaps substantially into the Common Era. Prior to the British Raj in 1858 the only evidence of military interaction with other regions is the rather ill-fated expedition of Alexander the Great. Here we take the end data to be collapse of Mogul rule in 1748.

Possibility of hegemony in the core. The Moguls ruled over India starting in about 1526. Prior to 1 CE the Maurya Empire held a hegemony over the subcontinent for several hundred years. During the era of interest there were several empires that briefly ruled over the bulk of the region: the Delhi Sultanate between 1321 and 1398 and the Kushan Empire under the rule of Kanishka I between 127 and 140.

Evidence for a one way barrier. As the Himalayas now and then impose an impenetrable barrier at least as far as military action is concerned, the relevant outsiders are the Southeast Asians in the East and Central Asians in the West. As we can find no record of any successful conquest or military intervention in Southeast Asia or by Southeast Asians, we conclude that the geographical barrier on the East isolates the two regions at least as far as military action is concerned. The situation on the West is one of classical outsiders: there is no record of any military action from India that made it past Kabul in Afghanistan. In the other direction, however, successful invasions from Central Asia into India have been common: we have the Kushans around 1 CE, the Huns around 400 CE, and beginning in the 600s a series of incursions by the Muslims

From the time Muslims started arriving, around 632 AD, the history of India becomes a long, monotonous series of murders, massacres, spoliations, and destructions. It is, as usual, in the name of “a holy war” of their faith, of their sole God, that the barbarians have destroyed civilizations, wiped out entire races. (Basu and Miroshnik (2017) pp.52 ff.)

This culminated with the successful invasion from the Central Asia by Babur in 1526 CE.

China

We take the core area of China to be the current area of China minus the Mongolian region (the provinces of Heilongjiang, Inner Mongolia (Neimongol), and Xingjiang) and the periphery to be the Mongolian region. Specifically the boundaries of the core are the Gobi desert in the West, the Himalayan mountains in the South and the Mongolia region (including Mongolia) in the North.

Period. As the Han dynasty overlaps the beginning of the Common Era, we take the start of that dynasty in 202 BCE as our start date. For the end date the Opium Wars are the first significant military intervention in China by the European powers, so we take the beginning of that war in 1839 as our end date.

Possibility of hegemony in the core. The history of China is one of hegemony more often than not: in our data 60% of our observations are of hegemonic societies.

Evidence for a one way barrier. There are several potential peripheral areas of China: Southeast Asia, Korea and Japan and Mongolia. Korea and Southeast Asia fall into a peripheral category that is not relevant to the model: both have been successfully invaded and conquered from China, both have successfully regained their independence from China, but we can find no evidence of

important military intervention into the core of China. With Japan the only evidence we have of military action in either direction is the ill-fated adventure of Kubla Khan in 1281 that gave rise to the well known expression “kamikaze.” In this context it should be noted that while England is quite close both the European continent and the population centers there, Japan is relatively far from the Asian continent and more so from the population centers.

Mongolia is a different story. Never occupied by the Chinese, the interventions of the Mongols in China have been constant and well-documented. The building of the great wall specifically to keep the Mongols out, as well as the conquests of Genghis and Kublai Khan make this clear.

Europe and the Middle East

We take the core areas to be continental Europe, by which we mean Europe excluding Britain, the Scandinavian peninsula and Russia and the periphery to be Britain, the Scandinavian peninsula and Russia. Specifically the boundaries of the core are the sea, including the Atlantic, Mediterranean, English Channel, North Sea, Baltic Sea and Black Sea, along with the border with the former USSR, which is to say roughly the current eastern borders of Poland, Slovakia and Romania.

A problem of Europe. There are two distinct ways of mapping Europe and the Middle East (including North Africa and Persia) into the model: we can view them as two separate cores each an outsider with respect to the other, or we can view them as a single core. Each view has merits. The Romans had a hegemony over both regions (but not Persia); after the Roman period each region posed a threat to the other, but neither made substantial inroads into the core of the other region. Geographically it is the presence of the Mediterranean sea that makes the situation difficult: this posed both a barrier to invasion and a road to invasion. Moreover due to technological change and migration the location of the periphery has changed: Germany which was clearly peripheral in Roman times was not so subsequently.

Fortunately for the purposes of the model it does not make a great deal of difference which point of view we take. For the bulk of the period, from the division of the Roman empire in 330 CE until the fall of Constantinople in 1453 CE the only important incursion from the Middle East into Europe was the prolonged Islamic occupation of Spain. After about 800 CE the various Islamic principalities in Spain were largely independent of North Africa - that is, European, not Middle Eastern - and did not in any case make incursions outside of Spain. After 1453 CE the Ottomans ruled over the Balkans but after their defeat in Vaslui in 1475, they did not pose a threat to the core areas of Europe. Hence we will take the view that the core area is what we will call Europe - which is to say Western and Eastern Europe excluding Britain and the Scandinavian peninsula. We will take the periphery to be Britain, the Scandinavian peninsula and Russia.

Period. The start date clearly lies with the Roman hegemony. There are two sensible start dates: the fall of Carthage in 149 BCE and the effective fall of the Republic with the crossing of the Rubicon in 49 BCE. It makes no difference to the analysis which we choose and we took the fall of Carthage as the start date. The end date is also unclear as other regions even today have not

had a military impact on Europe. Rather it makes sense to choose the end date the point at which Europe began to look outwards rather than inwards for conquest. Any such date is arbitrary: we took the end date as the date of the settlement of Jamestown, the first permanent settlement in North America. If we took the end date a century earlier or later it would not change the empirical analysis.

Possibility of hegemony in the core. During the Common Era hegemony has not been common in Europe, but it was achieved by the Romans (excluding Germany), nearly achieved, albeit briefly, by the Holy Roman Empire, and even more briefly by Napoleon and Hitler in the modern era.

Evidence for a one way barrier. Britain was occupied indeed by the Romans, but the improvement in British naval technology made Britain immune from serious invasion from the continent from 1066 on. The Swedes ruled over various parts of Europe but were never successfully invaded by the Europeans. Similarly there have been successful Russian interventions into Europe, but to date there has been little success going the other direction.

Strength of Outsiders

We measure the strength of outsiders by the ratio of the population of peripheral areas to the core. Except as otherwise indicated data is from Maddison (2013). In our initial evaluation of population we keep fixed the periphery as Europe as Sweden, Norway and the UK. We start with raw figures population from Maddison in millions:

Table 2: Population.

region	1 CE	1000 CE	1500 CE
SNU	1.1	2.6	5
Europe	30	30	61
Japan	3	7.5	15.4
China	59	59	103
Persia	4	4.5	4
India	75	75	110

SNU is Sweden, Norway and the UK; Europe is Western plus Eastern Europe less SNU. As there is no historical population data for either Central Asia or the Mongolian region the idea is to use Persia as a proxy for Central Asia and Japan as a proxy for the Mongolian region. Using Maddison (2013) from 2000 we define the Central Asian region to be the former Soviet central Asian republics (the “stans”) Azerbaijan, Kazakhstan, Kyrgyzstan, Tajikistan, Turkmenistan and Uzbekistan plus Afghanistan which had a combined population of 98 million as against an Iranian population of 66 million. Hence we estimate the population of Central Asia to be 1.5 times that of Persia (Iran). From the State of China Atlas 2009 (see <http://www.sacu.org/provtable.html>) we find the 2000 population of the “Mongolian” provinces of China are Heilongjiang 38, Inner Mongolia (Neimongol) 24, Xingjiang 21, while the population of Mongolia itself (from Maddison) is 3, so that the combined

population is 84 against a Japanese population of 127. Hence we estimate the population of the Mongolian Region to be two thirds that of Japan. We make one exception to this: the population of Japan more than doubled between 1000 CE and 1500 CE. Given the huge exodus documented in the text around 1200-1300 it is more likely that the population of the Mongolian region declined. As a conservative estimate we take the population of the Mongolian region in 1500 CE to be the same as in 1000 CE - that is 2/3rd of the Japanese population in 1000 CE.

The demographic record of the first half of the Common Era is marked by what we would call early globalization: the rising population of peripheral areas relative to the core areas of civilization (Western Europe, China and India). The table below records the increase in peripheral population relative to core population in the three regions.

Region	1 CE	1000 CE	1500 CE
India	8.0%	9.3%	5.5%
China	3.4%	8.5%	5.0%
Europe	3.7%	8.7%	8.2%

Based on this data we see a strong gap between weak outsiders: 3.4-5.5% outsiders and strong outsiders: 8.0-9.3% outsiders, and we classify outsiders as weak and strong accordingly.

This limited data is consistent with what we know more broadly about the central Asian diaspora of 200-400 CE: the massive movement of Germanic peoples from Central Asia to Northern Europe around 200 CE is well known - and their influence in the decline and fall of the Roman Empire well documented. In China a similar event occurs with the arrival of the “five barbarians” around 300-400 CE while in the 400s the Alchon Huns arrived in India from Central Asia. It is probably accurate to say that population centers that had been largely separate (Central Asia, Europe, China, India) began to collide as population grew in all of these areas.

Notice that we have measured in Europe the population of outsiders by Sweden, Norway and the UK. In the early period these areas are less important than Germany, and in the later period Russia rises in significance. However: the key point is that the Germans were not an important influence during the height of the Roman era, and if outsiders were even stronger than we estimate in the later era it only reinforces our conclusions.

Strength of Fortifications

Siege technology for China and Europe is discussed in the text. For India gunpowder and cannons seem to arrived only with the Moguls: according to Baburnama, Habib and Baipakov (2004) sometime after 1496.

Geopolitics: Hegemony

Is there a hegemony or balance of power? Here we take balance of power to mean “not a hegemony” - in reality there were often more than two powers. While the model does not allow for more than two societies competing, in the absence of hegemony there was typically a competition between two alliances even if each alliance was made up of independent political entities.

India. Until the Mogul conquest of India in 1526 of the many Empires that came and went on the subcontinent the closest to a hegemony there is record of were the Kushan Empire under the rule of Kanishka I between 127 and 140 and Delhi Sultanate between 1321 and 1398. Following the Mogul conquest in 1526 India was hegemonized for the first time in the Common Era and - depending on how one counts Pakistan and Bangladesh - has remained one since.

China. The records for China are well established. The Han dynasties were a hegemony that began in 206 BCE and lasted through 220 CE. From 220 CE - 581 CE there was a period of warlordism that was most certainly not a hegemony. This is followed in 581 CE - 907 CE by the hegemonic Sui and Tang dynasties. In 907 CE - 960 CE there were the Five Dynasties and Ten Kingdoms which as the name suggests did not constitute a hegemony. This was followed by Song dynasty in 960 - 1279 CE which also was not a hegemony: the Song only ruled over part of China: it was in a balance of power with a powerful northern state: initial ruled by the Liao and subsequently by the Xia and Jin. After 1279 CE China reverted again to hegemony with the Yuan, Ming and Qing dynasties.

Europe. During the period 1 CE - 330 CE the Roman Empire was a hegemony. After the Empire was split in half in 330 CE the closest there was to a hegemony in Europe prior to 1550 CE was during the brief reign of Charlemagne.

Inclusiveness

For our evaluation of inclusiveness in China and Europe see the text: here we briefly discuss India which we have judged extractive throughout the historical period in question - finding no evidence for a period or periods of inclusiveness. Although the caste system waxed and waned it existed throughout the period and the superiority of the warrior castes over the merchant castes is strong evidence of extractiveness. Moreover, throughout the era India sported a medieval type of political organization in which local princes together with their warriors ruled over local districts.

Web Appendix

Shared Political Power

We first study the situation where in an inclusive hegemony a weighted average $(1 - \omega)y_m + \omega y_e$ of the gain to each group represents the “overall gain” to rebellion, and likewise in an extractive hegemony $\omega y_m + (1 - \omega)y_e$ represents the “overall gain.” Instead of assuming $\omega = 0$ as in the basic model assume only that $\omega < 1/2$.

Theorem. *The structure of circuits is the same as in the basic model and it remains the case that only ss , ss and ww can be stochastically stable. The cutoff for an extractive hegemony over an extractive balance of power is $\eta^*(\varphi, \omega)$ which is larger than the η^* of the basic model. When $\tau = 0$ and $\varphi < 1$ then ww has a greater modified radius than ss ; when $\varphi = 1$ then ss has greater modified radius than ww .*

Proof. We modify the resistance table used in the proof of theorem 1 to account for utility of rebels that is a weighted average of that of the commercial elites and that of the military elites. As before we indicate first the transition then the benefit from the transition to the commercial elites and military elites respectively. This is divided into two columns for occupier or occupied where as in the text, if the initial state is a hegemony these are the hegemony’s four groups; in case the initial state is a balance of power occupier versus occupied refers to the situation after transition. The difference is that now for example in the transition $s \rightarrow sw$, we had $y_a = \tau$ - the elite’s gain - we now have the average $(1 - \omega)y_e + \omega y_m = (1 - \omega)\tau + \omega(1 - \tau)$. As before the asterisks denote which group is making the decision about attack and paying for defense and the final column reports the resistance.

transition	occupier	occupied	$r(z \rightarrow z')$
$ss \rightarrow s$	$0, 1^*$	$0, -1^*$	$U - \Pi_b(\eta) - u(1, 1, \varphi)$
$s \rightarrow ss$	$0, -1^*$	$0, 1^*$	$U - \Pi_h(\eta)(1 - \omega) - u(1 - \omega, 1, \varphi)$
$s \rightarrow sw$	$0, -1^*$	$1 - \tau, \tau^*$	$U - \Pi_h(\eta)((1 - \omega)\tau + \omega(1 - \tau)) - u((1 - \omega)\tau + \omega(1 - \tau), 1, \varphi)$
$sw \rightarrow s$	$0, 1^*$	$-(1 - \tau)^*, -\tau$	$U - \Pi_b(\eta) - u(1, 1 - \tau, \varphi)$
$sw \rightarrow w$	$0^*, 1$	$0, -1^*$	$U - u(0, 1, \varphi)$
$w \rightarrow sw$	$0^*, -1$	$0^*, 1$	$U - \Pi_h(\eta)\omega - u(\omega, 0, \varphi)$
$w \rightarrow ww$	$0^*, -1$	$1 - \tau^*, \tau$	$U - \Pi_h(\eta)((1 - \omega)(1 - \tau) + \omega\tau) - u((1 - \omega)(1 - \tau) + \omega\tau, 0, \varphi)$
$ww \rightarrow w$	$0^*, 1$	$-(1 - \tau)^*, -\tau$	$U - u(0, 1 - \tau, \varphi)$

We again start by determining the structure of circuits. We observe that

$$\begin{aligned} r(s \rightarrow sw) &= U - \Pi_h(\eta)((1 - \omega)\tau + \omega(1 - \tau)) - u((1 - \omega)\tau + \omega(1 - \tau), 1, \varphi) \\ &> U - \Pi_h(\eta)(1 - \omega) - u(1 - \omega, 1, \varphi) = r(s \rightarrow ss) \end{aligned}$$

for $\omega < 1 - \omega$ by the monotonicity property of $u(y_a, y_d, \varphi)$. This implies that ss and s form a circuit. Also by monotonicity we have $r(w \rightarrow ww) = U - \Pi_h(\eta)((1 - \omega)(1 - \tau) + \omega\tau) - u((1 - \omega)(1 - \tau) + \omega\tau, 0, \varphi) < U - \Pi_h(\eta)\omega - u(\omega, 0, \varphi) = r(w \rightarrow sw)$ for $\omega < 1 - \omega$ so that ww and w

form a circuit. Since the resistance at a balance of power is unchanged the resistance of joining the $ss \leftrightarrow s$ circuit is still less than joining the $w \leftrightarrow ww$ circuit. Hence circuits are still described by the diagram $[(ss \leftrightarrow s) \leftrightarrow sw] \leftrightarrow (w \leftrightarrow ww)$.

We next revise the table of modified resistances. The second column shows how the modified radius depends upon the resistances, the third how they depend on the model parameters. In the computation we make use of the fact that $u(0, y_d, \varphi) = 0$.

state	R	R
ss	$r(ss \rightarrow s) - r(s \rightarrow ss) + r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s)$	$\Pi_h(\eta)(1 - \omega) + u(1 - \omega, 1, \varphi) - \Pi_b(\eta) - u(1, 1, \varphi) + R(s)$
s	$r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s)$	$U - \Pi_h(\eta)((1 - \omega)\tau + \omega(1 - \tau)) - u((1 - \omega)\tau + \omega(1 - \tau), 1, \varphi) + \Pi_b(\eta) + u(1, 1 - \tau, \varphi)$
sw	$r(sw \rightarrow w)$	U
w	$r(w \rightarrow sw)$	$U - \Pi_h(\eta)\omega - u(\omega, 0, \varphi)$
ww	$r(ww \rightarrow w) - r(w \rightarrow ww) + r(w \rightarrow sw)$	$U + \Pi_h(\eta)((1 - \omega)(1 - \tau) + \omega\tau) + u((1 - \omega)(1 - \tau) + \omega\tau, 0, \varphi) - \Pi_h(\eta)\omega - u(\omega, 0, \varphi)$

As before $R(ww) > R(w), R(sw)$ so these two states are not stochastically stable. We compare

$$R(ss) - R(s) = \Pi_h(\eta)(1 - \omega) + u(1 - \omega, 1, \varphi) - \Pi_b(\eta) - u(1, 1, \varphi).$$

We see then that $\eta^*(\varphi, \omega)$ is the unique solution from equating this to zero. Notice that

$$\Pi_b(\eta)(1 - \omega) + u(1 - \omega, 1, \varphi) - \Pi_b(\eta) - u(1, 1, \varphi) < 0$$

so that $\Pi_h(\eta^*(\varphi, \omega)) - \Pi_b(\eta^*(\varphi, \omega)) > 0$ implying that fewer outsiders are needed to tip towards a balance of power than in the basic model.

Next consider $\tau = 0$. We have

state	R	R
ss	$r(ss \rightarrow s) - r(s \rightarrow ss) + r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s)$	$\Pi_h(\eta)(1 - \omega) + u(1 - \omega, 1, \varphi) - \Pi_b(\eta) - u(1, 1, \varphi) + R(s)$
s	$r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s)$	$U - \Pi_h(\eta)\omega - u(\omega, 1, \varphi) + \Pi_b(\eta) + u(1, 1, \varphi)$
ww	$r(ww \rightarrow w) - r(w \rightarrow ww) + r(w \rightarrow sw)$	$U + \Pi_h(\eta)(1 - \omega) + u((1 - \omega), 0, \varphi) - \Pi_h(\eta)\omega - u(\omega, 0, \varphi)$

from which

$$\begin{aligned} R(ss) - R(ww) &= \Pi_h(\eta)(1 - \omega) + u(1 - \omega, 1, \varphi) - \Pi_b(\eta) - u(1, 1, \varphi) \\ &\quad - \Pi_h(\eta)\omega - u(\omega, 1, \varphi) + \Pi_b(\eta) + u(1, 1, \varphi) \\ &\quad - \Pi_h(\eta)(1 - \omega) - u(1 - \omega, 0, \varphi) + \Pi_h(\eta)\omega + u(\omega, 0, \varphi) \\ &= u(1 - \omega, 1, \varphi) - u(\omega, 1, \varphi) - u(1 - \omega, 0, \varphi) + u(\omega, 0, \varphi). \end{aligned}$$

From $u(y_a, y_d, \varphi) = \alpha [(-\varphi - \beta(1 - \varphi)^2 y_d) y_a + (1/2)\beta y_a^2]$ we see that

$$R(ss) - R(ww) = -\alpha\beta(1 - \varphi)^2(1 - 2\omega) < 0.$$

Finally consider $\varphi = 1$. We re-display the three relevant rows for convenience:

state	R	R
ss	$r(ss \rightarrow s) - r(s \rightarrow ss) + r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s)$	$\Pi_h(\eta)(1 - \omega) + u(1 - \omega, 1, \varphi) - \Pi_b(\eta) - u(1, 1, \varphi) + R(s)$
s	$r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s)$	$U - \Pi_h(\eta)((1 - \omega)\tau + \omega(1 - \tau)) - u((1 - \omega)\tau + \omega(1 - \tau), 1, \varphi) + \Pi_b(\eta) + u(1, 1 - \tau, \varphi)$
ww	$r(ww \rightarrow w) - r(w \rightarrow ww) + r(w \rightarrow sw)$	$U + \Pi_h(\eta)((1 - \omega)(1 - \tau) + \omega\tau) + u((1 - \omega)(1 - \tau) + \omega\tau, 0, \varphi) - \Pi_h(\eta)\omega - u(\omega, 0, \varphi)$

To compute $R(ss) - R(ww)$ observe that the U terms and all the $\Pi_\zeta(\eta)$ terms cancel out. Hence

$$\begin{aligned} R(ss) - R(ww) &= u(1 - \omega, 1, \varphi) - u(1, 1, \varphi) - u((1 - \omega)\tau + \omega(1 - \tau), 1, \varphi) + u(1, 1 - \tau, \varphi) \\ &\quad - u((1 - \omega)(1 - \tau) + \omega\tau, 0, \varphi) + u(\omega, 0, \varphi) \end{aligned}$$

We have $u(y_a, y_d, \varphi) = \alpha [-y_a + (1/2)y_a^2]$. All the linear terms cancel out, so we compute all the quadratic terms, that is the sum of squared offensive elements S_a^2 . We get

$$\begin{aligned} S_a^2 &= -1 + (1 - \omega)^2 - ((1 - \omega)\tau + \omega(1 - \tau))^2 + 1 - ((1 - \omega)(1 - \tau) + \omega\tau)^2 + \omega^2 \\ &= 2\tau(1 - \tau)(1 - 2\omega)^2 \end{aligned}$$

which is positive since $\omega < 1/2$. \square

Extractive Cost Advantage

We suppose now that extractive and inclusive societies $k \in s, w$ have different costs of raising resources for conflict γ_k with $\gamma_w > \gamma_s > 1$. From the proof of Theorem 2 we have the optimum for the aggressor $\hat{x}_a = \alpha y_a / \gamma_a$ and for the defender is $\hat{x}_d = \alpha(1 - \varphi) y_d / \gamma_d$. Hence the partial utility gain to the aggressor is given by

$$u(y_a, y_d, \varphi, \gamma_a, \gamma_d) = \alpha [(-\varphi - \alpha(1 - \varphi)^2 y_d / \gamma_d) y_a + (1/2) \alpha y_a^2 / \gamma_a]$$

and note that this is increasing in γ_d and decreasing in γ_a .

Theorem. *The structure of circuits is the same as in the basic model and it remains the case that only s, ss and ww can be stochastically stable. The modified radius of s is remains greater than ss if and only if $\eta < \eta^*$. For $\tau = 0, \varphi = 0$ the modified radius of ww is greater than ss if and only if $\gamma_w < 3\gamma_s$.*

Proof. The resistance table used in the proof of theorem 1 is now

transition	occupier	occupied	$r(z \rightarrow z')$
$ss \rightarrow s$	$0, 1^*$	$0, -1^*$	$U - \Pi_b(\eta) - u(1, 1, \varphi, \gamma_s, \gamma_s)$
$s \rightarrow ss$	$0, -1^*$	$0, 1^*$	$U - \Pi_h(\eta) - u(1, 1, \varphi, \gamma_s, \gamma_s)$
$s \rightarrow sw$	$0, -1^*$	$1 - \tau, \tau^*$	$U - \Pi_h(\eta)\tau - u(\tau, 1, \varphi, \gamma_w, \gamma_s)$
$sw \rightarrow s$	$0, 1^*$	$-(1 - \tau)^*, -\tau$	$U - \Pi_b(\eta) - u(1, 1 - \tau, \varphi, \gamma_s, \gamma_w)$
$sw \rightarrow w$	$0^*, 1$	$0, -1^*$	U
$w \rightarrow sw$	$0^*, -1$	$0^*, 1$	U
$w \rightarrow ww$	$0^*, -1$	$1 - \tau^*, \tau$	$U - \Pi_h(\eta)(1 - \tau) - u(1 - \tau, 0, \varphi, \gamma_w, \gamma_w)$
$ww \rightarrow w$	$0^*, 1$	$-(1 - \tau)^*, -\tau$	U

We again examine the structure of circuits. We observe that $r(s \rightarrow sw) = U - \Pi_h(\eta)\tau - u(\tau, 1, \varphi, \gamma_w, \gamma_s) > U - \Pi_h(\eta) - u(1, 1, \varphi, \gamma_s, \gamma_s) = r(s \rightarrow ss)$ by the monotonicity property of

$u(y_a, y_d, \varphi, \gamma_a, \gamma_d)$. This implies that ss and s form a circuit. Also by monotonicity we have $r(w \rightarrow ww) = U - \Pi_h(\eta)(1-\tau) - u(1-\tau, 0, \varphi, \gamma_w, \gamma_w) < U - u(0, 0, \varphi) = r(w \rightarrow sw)$ so that ww and w form a circuit. We check which circuit sw joins. It will join the circuit to which it has the least resistance of reaching. By monotonicity $r(sw \rightarrow s) = U - \Pi_b(\eta) - u(1, 1-\tau, \varphi, \gamma_s, \gamma_w) < U = r(sw \rightarrow w)$ so the resistance of joining the $ss \leftrightarrow s$ circuit is less than joining the $w \leftrightarrow ww$ circuit. The circuit diagram: $[(ss \leftrightarrow s) \leftrightarrow sw] \leftrightarrow (w \leftrightarrow ww)$ is unchanged.

For the modified radii we have

state	R	R
ss	$r(ss \rightarrow s) - r(s \rightarrow ss) + r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s)$	$U + \Pi_h(\eta)(1-\tau) + [u(1, 1-\tau, \varphi, \gamma_s, \gamma_w) - u(\tau, 1, \varphi, \gamma_s, \gamma_s)]$
s	$r(s \rightarrow sw) + r(sw \rightarrow w) - r(sw \rightarrow s)$	$U + \Pi_b(\eta) - \Pi_h(\eta)\tau + [u(1, 1-\tau, \varphi, \gamma_s, \gamma_w) - u(\tau, 1, \varphi, \gamma_s, \gamma_s)]$
sw	$r(sw \rightarrow w)$	U
w	$r(w \rightarrow sw)$	U
ww	$r(ww \rightarrow w) - r(w \rightarrow ww) + r(w \rightarrow sw)$	$U + \Pi_h(\eta)(1-\tau) + u(1-\tau, 0, \varphi, \gamma_w, \gamma_w)$

As before $R(ww) > U$ so neither sw nor w is stochastically stable. As before $R(ss) - R(s) = \Pi_h(\eta) - \Pi_b(\eta)$ so between ss and s stochastic stability is determined entirely by whether $\eta > \eta^*$ or $\eta < \eta^*$. The key comparison remains

$$R(ss) - R(ww) = u(1, 1-\tau, \varphi, \gamma_s, \gamma_w) - u(\tau, 1, \varphi, \gamma_s, \gamma_s) - u(1-\tau, 0, \varphi, \gamma_w, \gamma_w).$$

We now specialize to $\tau = 0, \varphi = 0$: this becomes

$$\begin{aligned} R(ss) - R(ww) &= u(1, 1, 0, \gamma_s, \gamma_w) - u(1, 0, 0, \gamma_w, \gamma_w). \\ &= (1/\gamma_s)(1/2)\alpha^2 [\gamma_w/\gamma_s - 3] \end{aligned}$$

and we see that ww is stochastically stable over ss if and only if $\gamma_w < 3\gamma_s$. \square