Bayes Law

$$\mu(E|F) = \frac{\mu(F|E)\mu(E)}{\mu(F)}$$

“likelihood” times “prior”

Ratio Form

$$\frac{\mu(E|F)}{\mu(-E|F)} = \frac{\mu(F|E)\mu(E)}{\mu(F|-E)\mu(-E)}$$

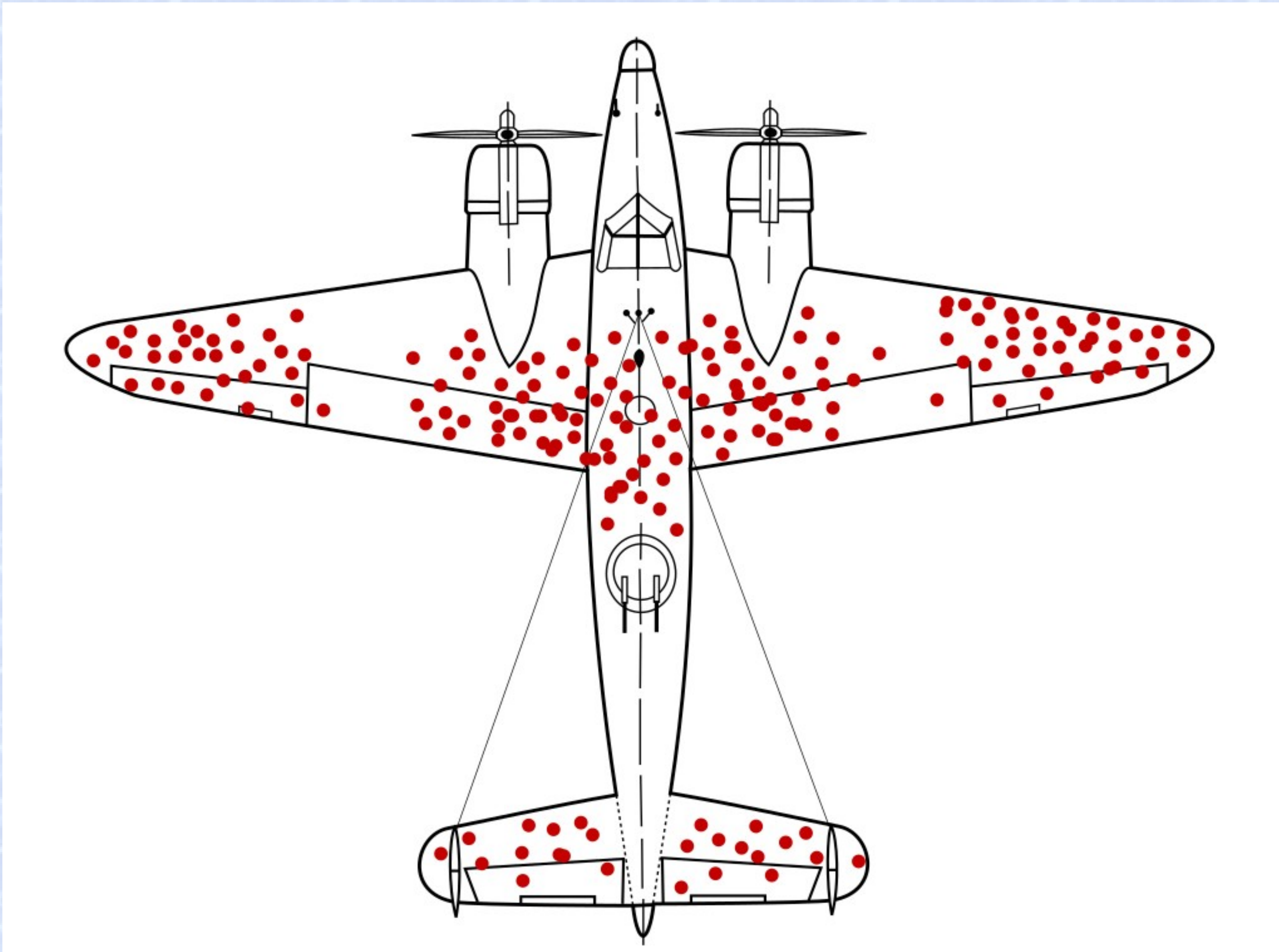
(Kahneman and Tversky)

Steve is very shy and withdrawn, invariably helpful but with very little interest in people or in the world of reality. A meek and tidy soul, he has a need for order and structure, and a passion for detail. (F=Steve)

Is Steve more likely to be a librarian or a farmer? (E=farmer)

Important bit of information: there are about twenty times as many farmers as librarians

Sample Selection Bias and Why We Do RCTs



Using Bayes Law

$\Pr(\text{shot}|\text{return})$ what we see

$\Pr(\text{return}|\text{shot})$ what we want

Getting these two confused is a common source of error

Drug Testing

A drug test has a 5% chance of error. A group of parolees is given the test. Of the parolees, 60% are drug users. If the test is positive how likely is it the parolee is using drugs?

E=using drugs

F=positive test

$$\mu(E|F) = \frac{\mu(F|E)\mu(E)}{\mu(F)}$$

$$= \frac{.95 \times .6}{.95 \times .6 + .05 \times 4} = .97$$

Airline Pilots

Now the test is given to a group of airline pilots of whom only 2% are drug users. If the test comes out positive how likely is it the pilot is using drugs?

$$\mu(E|F) = \frac{.95 \times .02}{.95 \times .02 + .05 \times .98} = .28$$

The Ann Landers Problem

Ann Landers says that all heroin users once used marijuana, so that if you use marijuana, you will surely end up using heroin

E=heroin use

F=marijuana use

$$\mu(E|F) = \frac{\mu(F|E)\mu(E)}{\mu(F)} = \frac{\mu(E)}{\mu(F)}$$

so that if there are 100 times as many marijuana users as heroin users, using marijuana means only a 1% chance of using heroin

Independence

We say two events E, F are independent if

$$\mu(E \cap F) = \mu(E)\mu(F)$$

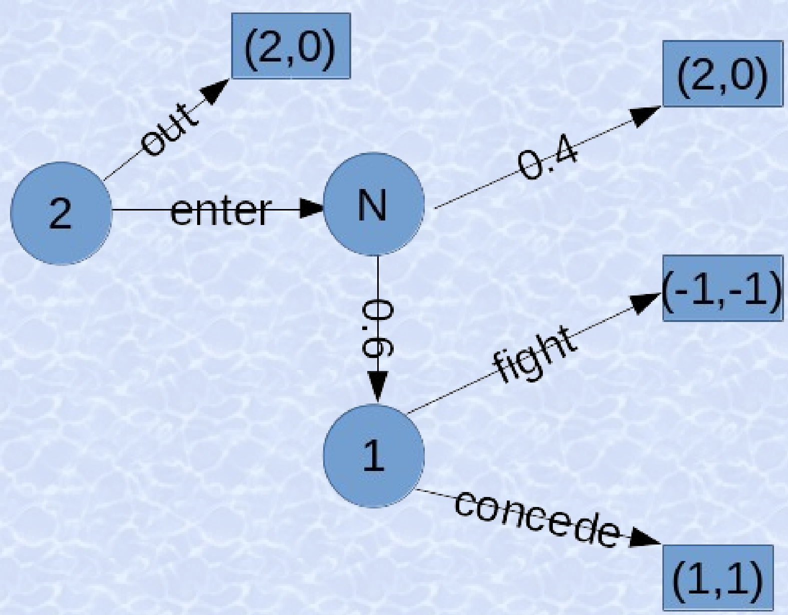
What is the conditional probability when events are independent?

$$\mu(E|F) = \mu(E \cap F)/\mu(F) = (\mu(E)\mu(F))/\mu(F) = \mu(E)$$

Nature's Moves

Add an additional player "Nature" with random moves

Example: Chain Store in declining industry



Decision Analysis

To drill for oil or not to drill for oil? Cost $100,000.

How much will you pay for a geological survey before drilling?

Value of Oil:

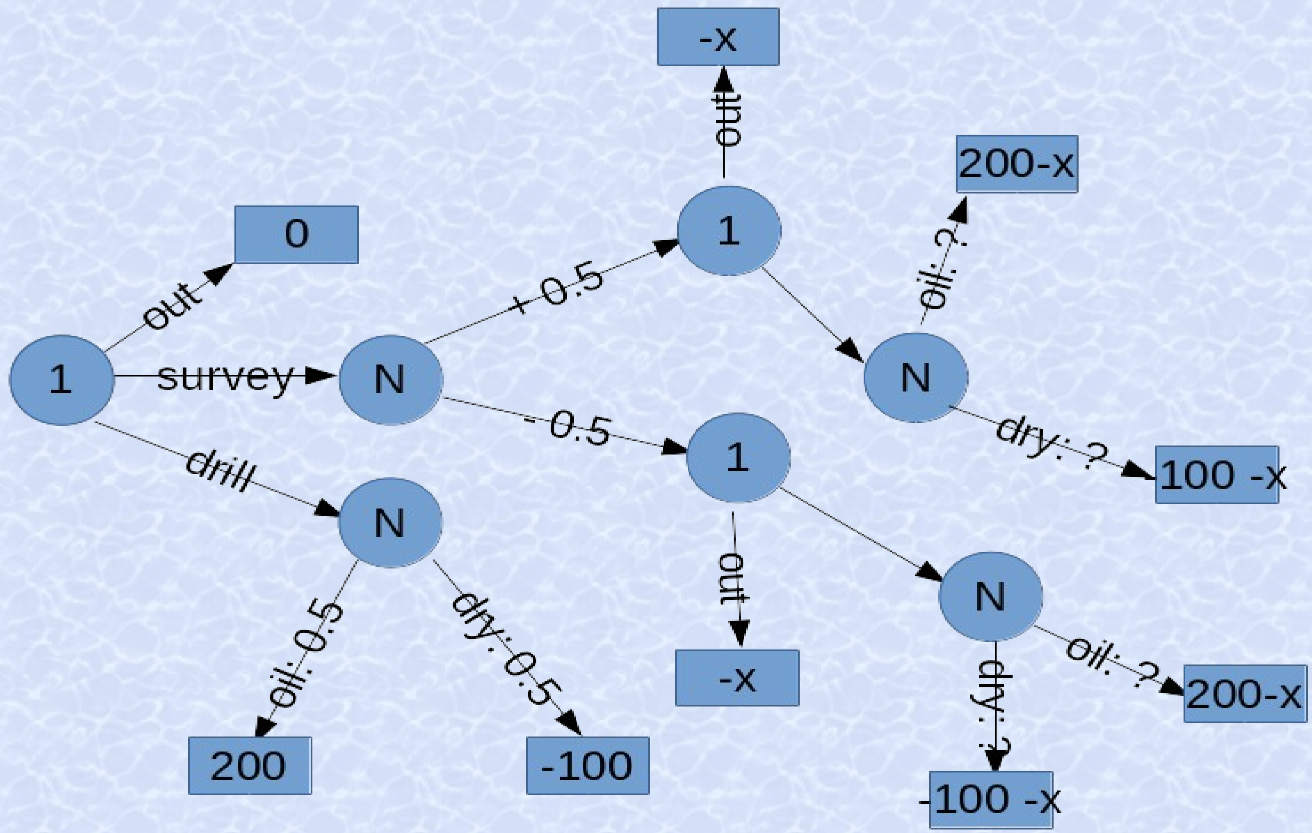
$0 (dry) with probability 50%

$300,000 with probability 50%

The survey has a 10% error rate

no risk aversion

Decision Tree

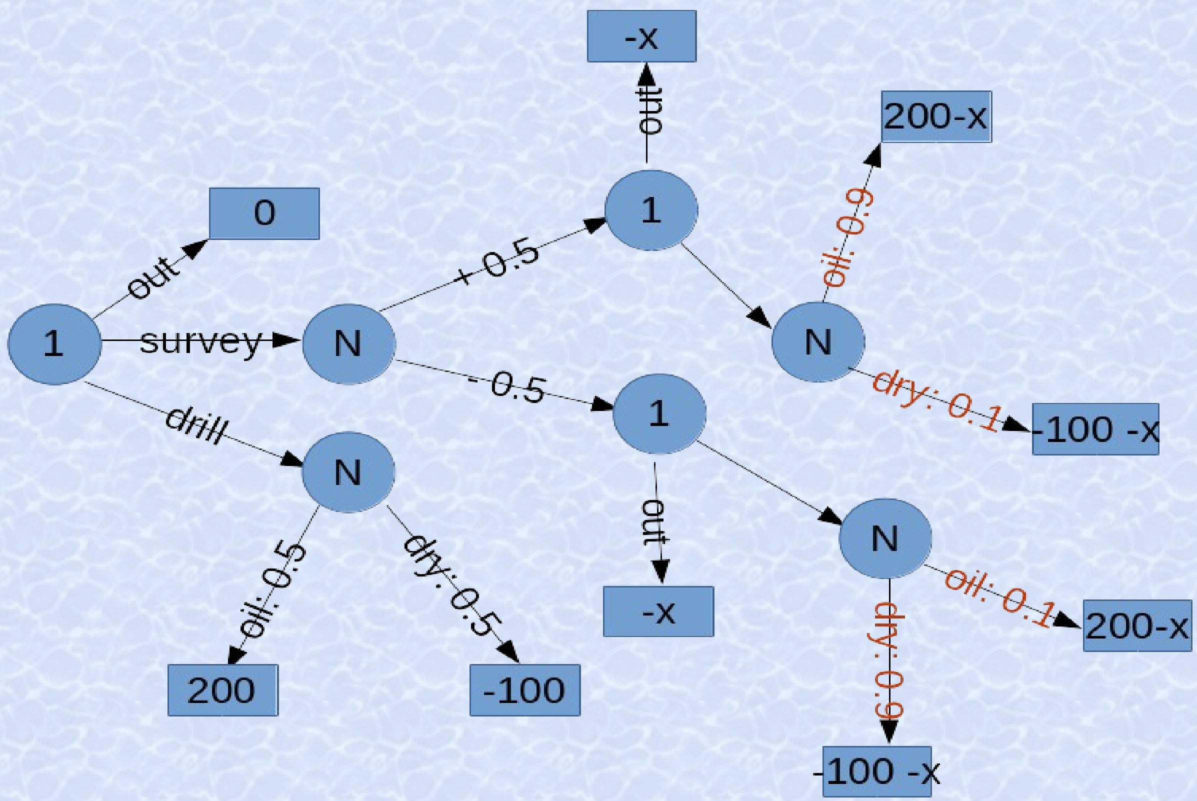


Where to Put Nature's Move

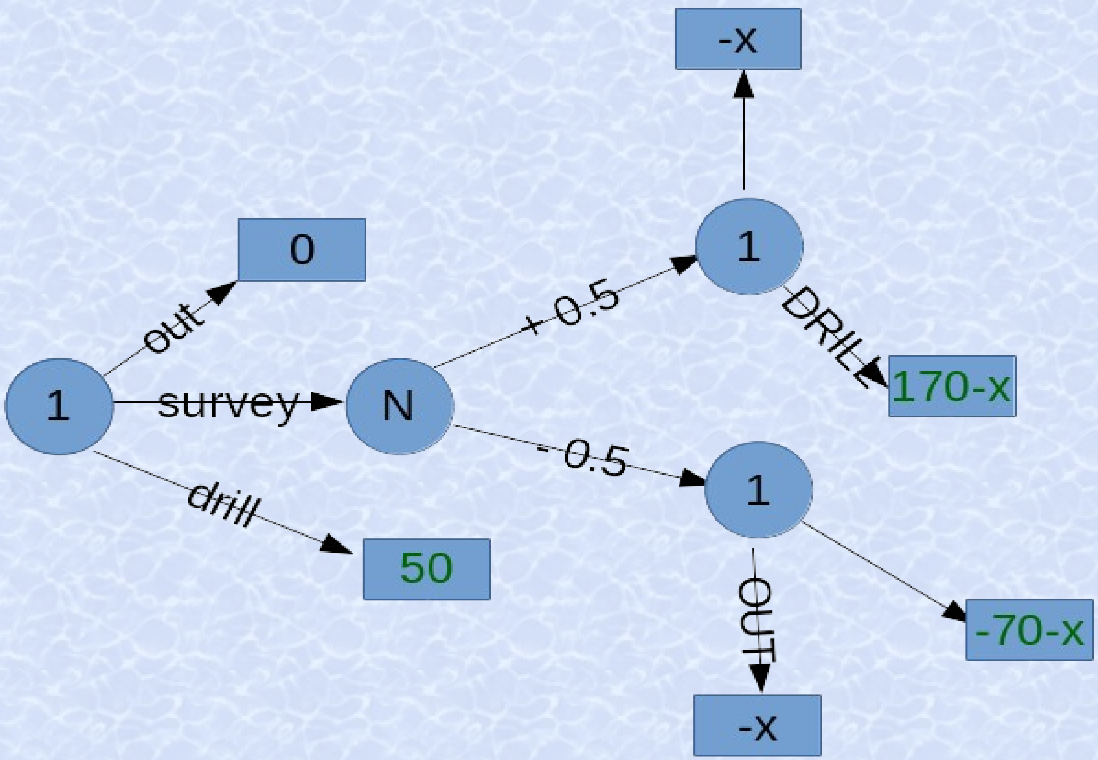
As late as possible to make it easier to do backwards induction

Filling in the ?'s

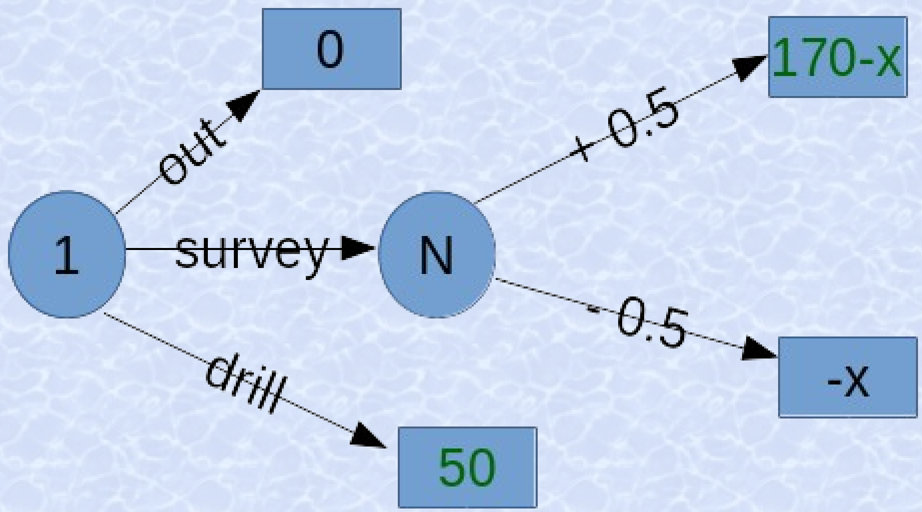
$$pr(dry|+) = \frac{pr(+|dry)\,pr(dry)}{pr(+)} = \frac{.1\times.5}{.5} = .1$$



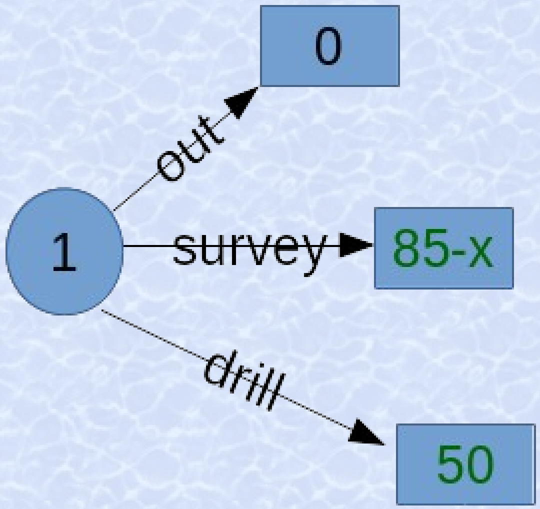
Compute Expected Utilities



Optimize



Expected Utility Again



drill or survey; survey if $85 - x > 50$ or $x < 35$

First Rule of Decision Analysis

- Do not pay for information that will not change your decision
- widely violated during Covid vaccination pauses

Concepts

- Conditional probability
- Bayes law
- independence
- value of information
- first rule of decision analysis

Skill

given information about conditional probabilities

do a cost/benefit analysis